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about this tutorial

Our aim is to help you understand the major elements that affect power quality: particularly inductive loads, harmonic distortion, and three-phase power unbalance. This will help you to solve the power problems you meet day to day in your work, now that electrical loads in industrial and commercial buildings are often too complex to be calculated by the Classical model of power.

The Classical (arithmetical) power model has been fine for over 100 years. For 80% or 90% of buildings and industrial processes, it still is. Over the last 10 years, though, there has been a fundamental shift from mainly resistive loads like heating elements and incandescent lights to low-energy lights and computers, and an increase in the number of motors and motor drives. That has increased inductive loads, harmonics and unbalance (current into the neutral line). The quality of power – its “fitness” to supply electrical equipment – has become increasingly important. Loads drawing currents with large inductive and harmonic elements can pollute the mains power supply voltage for other consumers. Polluting the power supply leads to utility penalty charges, costly wasted energy and even lost production. Fluke often comes across factories where mains pollution causes intermittent lock-ups and resets, corrupted data, premature equipment failure, and overheating of components for no apparent cause.

The basis of electrical and magnetic theory developed gradually until the 1870s, when Maxwell wrote down the basic equations of electricity and magnetism. Then, a period of only 20 years (often called the ‘second industrial revolution’) saw rapid development of practical electrical power transmission. Since the 1890s, we have seen a gradual refining of the theory.

After around a hundred years of discussion on how electrical power should be calculated under conditions of harmonics and unbalance, the IEEE 1459 power standard finally collects the definitions and mathematical approach into a single document. It is a great advance over the Classical method, and gives a ‘black box’ mathematical framework to calculate power factor. It gives you a single figure representing power quality, corresponding to the single figure the utility sees when looking into your (single or three-phase) power line.

A slightly different approach is taken by the Unified Power Measurement (UPM) model developed over the last 15 years by Professors Vincente Leon and Joaquín Montañana of the Universitat Politècnica de València, Spain. This approach attempts to unify the various AC power theories (hence the name). It extends Steinmetz’s theory (1897) – which showed that AC waveforms can be thought of as complex quantities – to all types of power systems, so with harmonics and unbalance.

The Unified model is elegant and comprehensive. It nicely complements IEEE 1459 by separating out the single utility power quality figure into corresponding physical elements that appear within your facility. Most importantly, that enables you to troubleshoot your power systems. Fluke has written algorithms separating out the different losses to give a powerful troubleshooting tool, with the Fluke 430 Series II Power Quality and Energy Analyzer incorporating both IEEE 1459 and Unified measurement methods. It calculates power quality by identifying the various power components so you can decide which to correct, and in what order. And that helps you save money on your energy costs and can also reduce your CO₂ footprint. Note that these savings are separate from the improvements in energy efficiency that can be made in buildings and processes. Here, we deal only with improvements related to power quality.

Until now, capturing the cost of energy waste caused by power quality issues was a task only for the most expert engineers. Waste costs could only be calculated by serious number crunching. The patented algorithms in the 430 Series II directly measure the waste caused by common power quality issues. To fully
benefit from the wealth of information the instrument provides, you need a basic understanding of how various powers tie together. With resistive and inductive loads introducing distortion and unbalance, things are not straightforward. Even if you can separate out the waveforms on each of the three phases, how do they translate into the system level powers? The utility supply powers all the individual pieces of equipment connected to the three phases, for example, but you receive a single system-level bill.

We’ll take it step by step, and you will gradually see active, reactive and other powers appear. We cover only the most practical power components. For the enthusiast there are many more, fully described in IEEE 1459 - 2010. That standard has two parts. The first collects the Classical equations into single and three-phase balanced and unbalanced, sinusoidal and non-sinusoidal power systems. The second describes Emanuel’s theory which says, among other things, that the effective apparent power of an unbalanced system (S_e) can be found by constructing a balanced power system with the same power losses as the actual system.

We cover all this in four Parts. Part 1 briefly covers DC power. Part 2 explains single-phase AC power starting with resistive loads, then adds inductive loads, and then harmonic distortion. Part 3 describes three-phase power, concentrating on the unbalance effects that occur when the loads are different on each of the three phases. It contrasts the Classical, IEEE 1459 and Unified methods (since this is where the different methods start giving different results). Part 4 introduces three-phase loss analysis and summarizes how to prevent the losses, and Part 5 is an appendix giving some more advanced technical material for the enthusiast. Throughout, we give dates for some of the important developments, summarized at the end in an electrical power timeline (part 6).
introduction to single-phase power

Before you can understand the three-phase power that is normally supplied to larger industrial and commercial buildings, you will need a thorough grounding in single-phase power. That means understanding the relationships between the sine-wave voltage supplied by your utility and the resultant currents and powers taken by the different types of equipment loads in your buildings.

There are four important forms of single-phase (two-wire) power.

Simplest is DC (direct current) power (section 1.1). At its most basic this is generated by the current from batteries to flashlights.

Basic AC (alternating current) power includes resistive loads like incandescent lamps and heating elements where voltage and current rise and fall together, so in phase (section 2.1). One question here is: how do you find the average power dissipated in a load from a continuously varying voltage?

While the relationship between voltage and current into a resistive load is relatively simple, things get more complicated with motors. These introduce a phase shift between current and voltage (section 2.2). When you increase the voltage across the coil of a motor, the current doesn’t immediately increase but lags behind it (remember ULI: U leads I in an inductor L; and ICE: I leads E in a capacitor C).

One consequence is that, for short times in the waveform, the voltage across the inductor is positive while the current is negative (and vice versa), which means that the power is briefly negative. So, besides taking power from the supply, the load briefly pushes power back into the supply during each cycle. This is the basic factor affecting “power quality”. Utilities do not like it because it can affect other users connected to the same line. Customers are therefore increasingly being charged for poor power quality.
Electronic devices and computers introduce harmonic distortion (section 2.3). Rather than dealing with simple sine waves, you then need to know how to describe distorted waves. These, too, can push power back into the supply every cycle and can also distort the voltage coming from the utility.

**Why should you be interested?**

In these panels we’ll summarize the key information to remember. We’ll also highlight the practical problems caused by each of the various effects, and broadly what you should do to compensate for them.

In general, the power losses generate heat in resistive loads and inductive loads (which normally also have a significant resistive component). In motors, the losses can also generate vibration and braking torque. They can mean that transformers need to be derated. And all of them can reduce equipment lifetimes.

Using capacitors in combination with a motor can correct for this wasted power, and save thousands of Euros a year. Similarly, filters may be needed to prevent distortion, and balancing equipment (or redistributing equipment across the phases) may need to correct unbalance.
1 DC power

1.1 basic DC power

We start with DC power for (say) a simple flashlight. DC is important to a power engineer because it includes the outputs of switched-mode power supplies that are found in many types of electronic equipment. DC is also still used in automotive and locomotive transport, and some motors. And, since the outputs of solar panels are DC, it is making something of a resurgence in importance.

Part of the work is to define the symbols used in IEEE 1459 (and elsewhere). Things get complex further on so, to understand exactly what is going on, it is important to get the basics right.

The diagram shows a 6 Volt battery driving 1 Amp into a 6 Watt incandescent lamp.

By convention, constant values are given capital letters, so DC voltages are given by a capital U, DC currents by capital I, and resistance by capital R (no subscript because DC and AC resistances are the same). The respective units are volts (V), amps (A) and ohms (Ω).

Ohm’s law

The basic equation governing voltage and current was first published in 1827 in a book written by Georg Ohm. The simple statement of \( U = I \cdot R \) started the whole subject of power metrology.

Power

DC power is represented by a capital P and measured in watts (W). When we need to be clear, we use subscripts to confirm that values are DC, but leave them off when it is obvious what is referred to.
Energy

Power is the rate of energy flow in a circuit. So, energy is the power multiplied by the time \((t)\) that it flows, generally measured in Watt hours (Wh) or kilowatt hours (kWh). If the power is not constant you can approximate it by taking many samples, each lasting a short time \(\Delta t\) over which the power \(\Delta P\) is near constant. Then, the energy is the sum of all the individual power samples. Mathematically, the energy is given by integrating all the instantaneous values of power for the time that it flows. From here, we will skip the integral operator and stick to samples (as for example a digital meter or Energy/Power Analyzer take). Either way though, even with a constant power, the energy changes constantly over time.

In the 1880s, Thomas Edison and Westinghouse Electric Corporation were engaged in a ‘Battle of the Currents’. Edison wanted DC power to supply homes and factories, while Westinghouse favoured AC. If Edison had won, that would almost have been the end of our discussion (the three-phase power that makes up most of this tutorial, for example, would bring little advantage). He didn’t, though, and AC won out because it is easier to generate the high voltages and so low currents that make distribution more efficient (power loss is \(I^2R\)), and because (synchronous) rotating machines are easier to drive with AC power.
### 2 single-phase AC power

#### 2.1 single-phase power with resistive loads

##### 2.1.1 resistive loads – basic AC power

For basic AC power into a resistive load, the voltage $U_{AC}$ across the load pushes a current $I_{AC}$ into the resistance $R$. The illustration this time shows a mains voltage of 120 V @ 60 Hz driving 5 A into a (powerful) 600 W lamp. This could be a stage spotlight or a ring circuit of thirty 20 W lamps, for example.

**Instantaneous voltage, current and power**

With AC, the voltage and so the current and power vary over time. The most general statement is that instantaneous power $p(t) = u(t).i(t)$ at any time.

Rotating generators have traditionally generated the voltage that supplies the current. When something rotates, the height of each point on the circumference varies as a sine wave. So a generator produces a sine-wave voltage (opposite over hypotenuse as the rotor rotates around the radius). Utilities are incidentally primarily interested in supplying a voltage – users take current from the supply but this shouldn’t influence the pure sine-wave voltage supply.

**Parameters**

We want to describe a waveform without going into all the complex time related details, and so we simplify by defining parameters. These are measurable properties of varying waveforms like $u(t)$, $i(t)$ and $p(t)$. But, for instantaneous values that are always changing, what do the figures 120 V and 5 A refer to? They are not the peak values, and they are not the average values (which for sinewaves would be zero). They generate an average power of 600 W in the load, though, giving the same heating effect as would a 600 W DC power.
The voltages and currents $U_{RMS}$ and $I_{RMS}$ are actually Root Mean Square values. We will now explain why these parameters are useful.
why RMS values?

Calculating with resistive load instantaneous values:

\[ u(t) = \hat{u}\sin(2\pi ft) \]
\[ i(t) = \hat{i}\sin(2\pi ft) \]
\[ p(t) = u(t) \cdot i(t) = \hat{u}\sin(2\pi ft) \cdot \hat{i}\sin(2\pi ft) \]
\[ P = \text{avg}[p(t)] \]

Instead of working with instantaneous values we look for simple parameters for power analogous to DC: \( P = U_{DC} \cdot I_{DC} \)

This is why we introduce \( U_{RMS} \) and \( I_{RMS} \) so for a resistive load we can say:

\[ P = U_{RMS} \cdot I_{RMS} \]

Note that \( u(t) \) and \( i(t) \) are signals (waveforms) that vary over time while \( U_{RMS}, I_{RMS} \) or simply \( U, I \) are parameters (numbers) representing \( u(t), i(t) \). We deal with the RMS values of AC voltage and current, but they give the average AC powers.

2.1.2 resistive loads – RMS values and why we use them

We can describe the value of \( u(t) \) using the peak value \( \hat{u} \) and the sine-wave function. The instantaneous value is now given by the peak value times the sine of the angle at any time \( t \) as it repeats at frequency \( f \) (2π radians = 360°, a complete rotation). Same for instantaneous current. Instantaneous power is, as before, \( p(t) = u(t) \cdot i(t) \) where \( p(t) \) is constantly fluctuating.

We now need a single figure to represent the average power that this varying voltage can deliver into a load to correspond with (develop the same heat as) a DC power \( P \). The average value of the instantaneous power \( p(t) \) similarly has parameter value \( P \).

We can’t however find this average power by multiplying the average values of voltage and current, because the average values of a sine wave are zero. We therefore need to get rid of the negative parts of the signal, and RMS is the most useful way of doing this. Separate out the instantaneous values of the varying voltage into \( n \) instants, and the RMS value is defined by

\[ U_{RMS} = \sqrt{\frac{u_1^2 + u_2^2 + \ldots + u_n^2}{n}} \]

– the square Root of the Mean (average) of the Squares.

For a pure resistive load like our lamp, voltage and current are in phase. This means the formula for power is in this case \( P = U_{RMS} \cdot I_{RMS} \). Here and elsewhere, RMS means that you don’t have to worry about the time function. To find the average power in a resistive load, you simply multiply \( U_{RMS} \) by \( I_{RMS} \).

Although the ‘root mean square’ is an abstract concept, it is very useful in power calculations because \( P = U^2/R = I^2R \), so the square of the values of \( u(t) \) and \( i(t) \) do represent something real – the heat loss across a resistor (squaring both sides of the equation above for \( U_{RMS} \) and dividing by \( R \) gives you the average power for all the \( n \) instants).
2.1.3 resistive loads – RMS voltages

We can look at the voltages as they could appear on an oscilloscope. The sine wave at the top repeats once every period (T). Rather than showing the time in seconds, we show the phase angle of $u(t)$, and again by convention divide the cycle into 360°. This can be drawn in an Excel program (drawing $u \cdot \sin \theta$ from 0 to 360°).

Squaring the instantaneous voltage (bottom diagram) prevents the negative values from simply cancelling out the positive ones, and also doubles the frequency. Power is now pushed into the load once every half cycle rather than every cycle – the current flows first one way and then the other but both times generates heat. Again, the power $P=U^2/R$.

We can find the relation between $\hat{u}$ and $U_{RMS}$ by looking at the square of the sine wave (bottom diagram again). For our 120 V RMS voltage, the average value $U^2$ is by definition $120^2$ and the peak-to-peak is $2 \times 120^2$. Taking the square root confirms that $U_{RMS}$ is 120 V, which means that the peak value $\hat{u}$ is $\sqrt{2} \times 120$ or 170 V. Once again, the 120 V $U_{RMS}$ is the voltage that will produce the same heat in a resistor as will a DC voltage of 120 V.

Values should in practice be taken over a complete period or, for better accuracy, over a number of complete periods. The standard IEC61000-4-30 prescribes 10 periods for 50 Hz or 12 periods for 60 Hz. A 60 Hz supply gives a period of 16.66 ms (the 50 Hz supplies in Europe and elsewhere have a period of 20 ms, but this makes no practical difference to the calculations).
2.1.4 resistive loads – RMS voltage properties

So, what are the basic properties of RMS voltages?

Adding RMS values

You can only add the RMS values of two signals directly if they have the same frequency and phase (as with two stacked transformer windings). If the two signals have the same frequency but are shifted in phase (so shifted in time) the resulting RMS value can vary from twice the RMS value of a single signal (when they are exactly in phase) to 0 (when they are exactly shifted half a period). In general, if you want to add the RMS values of voltages with different frequencies, you have to add the squared values and then take the square root. So a 10% harmonic voltage of 12 V AC added to a fundamental voltage of 120 V AC gives a total RMS value of $\sqrt{12^2 + 120^2} = 120.6$ V AC. So, a even just a 0.6 V change in RMS voltage can show 12-V harmonics.

RMS for a pure sine wave

For a pure sine wave (and only for a pure sine wave), the RMS voltage is the peak value over $\sqrt{2}$ (so again $170/\sqrt{2} = 120$ V).

Incidentally, don’t confuse the RMS value $U_{\text{RMS}}$ with $U_{\text{AVG}}$, the rectified average value. Although the average value of an AC signal is zero, rectify it and you get $U_{\text{AVG}}$, since full-wave rectifiers convert the negative values to positive ones to give the modulus $|u(t)|$. The RMS value is slightly (1.11 times) higher than the average. The old analog moving-coil voltmeters used to simply rectify a sine wave and multiply the average by 1.11 to show RMS. Again, this only works for a pure sine wave and otherwise gives incorrect results. So, average values are only ever of interest if you have a pure sine wave (or a very cheap multimeter.)

Parameters

The two key voltage parameters for a rectified sine wave are the peak and RMS values. Once again, you use the RMS value to find the power (heat) that voltage will develop.
2.1.5 resistive loads – voltage and current in phase

Multiplying the top two instantaneous voltage and current waveforms (in Excel again, for example) gives the bottom waveform – the instantaneous power. So, even though the averages of the RMS voltage and current are zero, the average power is positive – in itself an interesting result.

RMS voltage

For the sine-wave voltage \( u(t) \) shown, \( U_{RMS} \) is 120 V.

RMS current

Similarly for \( i(t) \), \( I_{RMS} \) is 5 A.

Multiplying them gives average power for a resistive load

The waveform at the bottom shows the instantaneous power, \( p(t) = u(t) \cdot i(t) \), which is always positive. The average power \( P = AVG[p(t)] \) (or mathematically \( P = 1/T.\int[p(t).dt] \)) is 600 W.

Note that the \( P \) stands for \( P_{AVG} \), and needs no subscript because \( U_{RMS} \) multiplied by \( I_{RMS} \) automatically gives the average power. So, \( P = U_{RMS}.I_{RMS} \) or simply \( P = U \cdot I \). This is true for a resistive load but not generally, as we shall now see.
2.2 single-phase power with inductive loads

2.2.1 inductive loads – introducing phase shift

This time we have an inductive load like a motor. Again, $I_{AC}$ and $U_{AC}$ are in capitals to show steady values. We’ve chosen the same 120 V\(_{\text{RMS}}\) supply pushing 5 A into a load for easy comparison with the resistive load. The motor is labeled 120 V, 600 VA and $\cos \phi = 0.866$, which will be explained in the next few pages.

The diagram shows the AC voltage and current into the motor’s stator coil. These produce a magnetic field that rotates around the coil, in turn inducing a rotating magnetic field in the rotor coil. The fields interfere to generate force, which turns the rotor. The principles are similar for a transformer, where the diagram shows the voltage across and current into the primary coil. The transformer’s secondary coil normally has either more windings than the primary and so steps up the voltage while decreasing the current, or fewer windings so steps down the voltage while increasing the current.

The diagram shows the effect of ‘looking into’ the stator coil or the transformer primary. The load is actually connected across the motor rotor (transformer secondary) coil but it is reflected back across the stator (primary) and so we only need to show the stator in the circuit.

**Instantaneous values**

As before, the instantaneous values are $u(t)$, $i(t)$ and $p(t)$ and the instantaneous power is $p(t) = u(t) \cdot i(t)$.

**Parameters**

The magnetic field set up by the coils forming an inductive load resists any sudden change in the current, which means that a change in voltage doesn’t immediately change the current: the current lags behind the voltage. That gives an extra parameter: the phase angle between voltage and current, $\phi_{UI}$.

So, what is the value of the power now?
2.2.2 inductive loads – voltage, current and power

We now again draw the voltage (top), current (centre) and power (bottom).

Voltage and current

We have the same RMS value of 120 V as before.

We again have a current of 5A, but this time notice the phase shift of -30° between voltage and current, minus because \( i(t) \) lags behind \( v(t) \) – it reaches its first zero crossing 30° after the first zero crossing of \( u(t) \).

For a pure inductor, the phase angle would be -90° (so the same amount of energy would flow back into the supply in the second half cycle as is transferred to the load in the first). Motor windings have both resistance and inductance, though, which alters the phase angle. In motors, the current can typically lag the voltage by 30° so we say \( \phi_{ui} = -30^\circ \) (notice the minus sign again). So how does this phase shift influence the power?

Instantaneous and average powers

Again, we have \( u(t), i(t) \) and the most general statement for the power of a waveform: \( p(t) = u(t) \cdot i(t) \).

And again, we can multiply the instantaneous values of voltage (top graph) and current (central graph) using an Excel spreadsheet to give the power as it varies over time. The period is half that of \( u(t) \) and \( i(t) \), so the frequency is 120Hz.

Twice in the cycle (when voltage is positive and current negative, and when voltage is negative and current positive) the value drops below zero, showing that power is pushed from the load back into the mains.

Now, the spreadsheet will show that the average power is no longer 600 W, but a lower figure of 520 W. The phase shift between voltage and current reduces the average of the power waveform by 80 W

So, for an inductive load the average power \( P \) is not equal to \( U_{\text{RMS}} \cdot I_{\text{RMS}} \).
2.2.3 inductive loads – separating instantaneous into active and reactive power

But how do you calculate the exact value of average power now that it is no longer simply $U_{\text{RMS}}I_{\text{RMS}}$? A mathematical trick suggested by Charles Steinmetz back in 1897 can separate out the instantaneous power $p(t)$ into two components. The first is the active power $p_a(t)$ – again a sine wave and with the same average as $p(t)$. It stays positive, though, and represents the power flowing into the resistive load. The second is the reactive power $p_r(t)$, which we choose to go positive and negative but which is 90° out of phase with the active power and has zero average power. These two form the instantaneous power at any time.

The power from the supply at any instant therefore separates out into the active power flowing into the resistive load plus (90° later) the reactive power flowing back and forth through the inductor.

So in the middle diagram the active power (green) drives the motor. The remainder is the reactive power (red) which flows into the load for half the time and flows back into the mains for the other half, with an average of zero. Although the reactive power generates no torque at the axle of motor, it still flows through the system and generates (wasted) heat.

The bottom diagram shows the two currents associated with these powers: the active current $i_a(t)$ which is in phase with the instantaneous voltage from the utility, and the reactive current $i_r(t)$ which is 90° out of phase. In effect, we are separating out the ‘real’ component of current through the resistive component of the motor (not just the winding resistance, note, but also to generate the mechanical power from the motor), and the ‘imaginary’ component at right angles to it which drives a current through the inductive component.

Once again, the same argument holds for a transformer. The input voltage and current are separated by the same phase angle as the output voltage and current, and the input power to the primary is (ignoring losses) the same as the output power from the secondary. Any resistive load connected to the secondary is reflected back to the primary, so the input side sees both inductance and resistance.
**Instantaneous power**

The instantaneous power is, again, $p(t)$ which for short times now goes negative.

**Active and reactive powers**

We have defined the instantaneous power to be the simple sum of the active and reactive powers at any time.

**Active and reactive current**

The instantaneous current is similarly the sum of active and reactive currents. Note that reactive power in itself does not cause loss, it is power that bounces between source and load but is not consumed. It does however produce reactive current which causes losses in the system (and load for the utility).

| The active power flows through the resistive part of the circuit and has the same average value as the instantaneous power, while the reactive power flows through the inductive part of the circuit 90° later and has an average value of zero. |
|---|---|
| The reactive power is a key measure when troubleshooting power systems, and to eliminate it you can compensate for the lagging current by adding a capacitor at the front end of a motor. The current through a capacitor leads the voltage, and the capacitor effectively supplies the reactive current to the motor locally so that the utility does not see it. |
2.2.4 inductive loads – active, reactive and apparent power

So, what are the parameter values for these instantaneous waveforms?

Parameters

Most of the time we’re not interested in instantaneous values of power but rather the average value \( P \), here 520 W. The average value of active power \( p_a(t) \) – the active power into the load – is also 520 W (which, since the average of the reactive power is zero, is how we constructed it).

Active and reactive powers

To recap, the instantaneous power (blue in the top diagram) is the total power flowing into the motor. We’ve broken this power down into an active power \( P \) (green in the middle diagram) and a reactive power \( Q \) (red). The active power is transformed into mechanical energy in a motor (or is transferred to the secondary circuit of a transformer). The reactive power does not produce any useful energy but just flows to and from the voltage source.

We know that the average of \( p_r(t) = 0 \), but this isn’t a good number to represent power. We therefore represent the reactive power \( p_r(t) \) by its maximum value. The reactive power \( Q = \max[p_r(t)] = 300 \text{ var} \). The new unit is Volt Amp Reactive, to indicate that we are dealing with reactive power.

Apparent power

Seen from the utility side, \( U_{\text{RMS}} \) and \( I_{\text{RMS}} \) are the parameters that determine the size of the network. For this reason the apparent power \( S \) of 600 VA, the simple product of 120 V \( U_{\text{RMS}} \) and 5 A \( I_{\text{RMS}} \), is used as a measure for the power in the network. The active power is then \( U_{\text{RMS}} \cdot I_{\text{RMS}} \cos\phi \) (120 * 5 * \cos30° = 600 * 0.866 = 520 W).
Apparent power is a measure of the power that has to be transported – the parameter value of the instantaneous power flowing through the network. It is important because it defines the size of the network (generators, transformers, lines). We give it the unit Volt Amps to distinguish it from real power in Watts and reactive power in vars – you have to treat them all separately. Active power in W is a real quantity, while reactive power in vars is imaginary and apparent power in VA is complex (strictly, the apparent power is the modulus of the complex power – the length of the vector). Understanding the relationship between active, reactive and apparent powers is fundamental to understanding power engineering.

You can add active powers to each other, and you can add reactive powers to each other, but you cannot add active to reactive powers because they act at right angles to each other. The apparent power – the power that appears at the utility – is actually the vector sum of P and Q (\(\sqrt{P^2 + Q^2}\)). (The English phrase ‘mind your Ps and Qs’ meaning ‘mind your manners’ might help as a reminder, but with electrical power the P’s and Q’s are easy – it’s the S you have to mind).

While active power is associated with the resistance R of a circuit (\(P=U^2/R\)), reactive power is associated with the reactance X of the circuit (\(Q=U^2/X\)) and apparent power with the impedance Z (\(S=U^2/Z\)).

At first sight, it looks strange using average values for P and S, but the peak value for Q. However, \(p_a(t)\) on the middle diagram varies around the average value 520 W, meaning that the peak value is also 520 W (peak-peak is 1040 W). Similarly S (not shown on the diagram, where it would appear as a straight line at 600 VA, above P) has an average and a peak value of 600 VA. We have defined the average value of Q as zero, but the peak is 300 var, and \(520^2 + 300^2 = 600^2\).

<table>
<thead>
<tr>
<th>To add or subtract active (or reactive) powers you can just add or subtract the instantaneous values because the powers act ‘along the same axis’. To add or subtract apparent powers you need the vector sum, so you have to first square the values and then take the square root.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Remember:</strong></td>
</tr>
<tr>
<td>active power: (P = \text{avg}[p_a(t)]) W</td>
</tr>
<tr>
<td>reactive power: (Q = \text{max}[p_r(t)]) var</td>
</tr>
<tr>
<td>apparent power: (S = U_{\text{rms}} \cdot I_{\text{rms}}) VA</td>
</tr>
<tr>
<td>and: (S^2 = P^2 + Q^2)</td>
</tr>
<tr>
<td>P, Q and S all have the same dimensions, but separating them into Watts, Volt Amps Reactive and Volt Amps reminds us that we can’t simply mix them. Another way to put it is (S = P + jQ), where (j) is the imaginary unit.</td>
</tr>
</tbody>
</table>

**Active and reactive currents**

Now \(i_a(t)\) and \(i_r(t)\) can be represented by their RMS values: \(I_a\) and \(I_r\). Because we are dealing with RMS values, the apparent current \(I^2 = I_a^2 + I_r^2\). We can again deduce the values – the RMS value of (green) active current \(I_a\) is simply 520 W/120 V = 4.33 A. The RMS value of (red) reactive current is similarly 300 var/120 V = 2.5 A, 90° out of phase.

This is another interesting result. Because of Pythagoras, a 30° phase shift from the motor gives a fairly large reactive component: 2.5 A is reactive against 4.3 A active for 5 A apparent. This 2.5 A does not contribute to moving the motor axle, but your network still has to carry it. And you may still have to pay for it, depending on how you are metered.
This separation of the apparent power (VA) into active (W) and reactive (var) powers is fundamental to power quality. You can compare it to pulling a canal boat.

Canal boats used to be pulled by horses walking along a towpath. The ‘apparent’ power is the power developed by the horse pulling the rope. The rope makes an angle \( \phi \) to the towpath, and you can separate out this apparent power into the real useful power that pulls the boat along the canal, and the reactive power that the rudder has to counteract just to prevent the barge being pulled into the side. Here, too, the powers are simply given by the vector sum \( S^2 = P^2 + Q^2 \).

Replace the horse by a utility generator, the rope by the network, and the boat by an electric load. The utility supplies the apparent power, the real power generates real useful torque in your motors, while the reactive power goes off into the imaginary direction and is wasted.

**The equations**

Much of what we’ve seen so far can be summarized in these three relationships:

<table>
<thead>
<tr>
<th>Power Type</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Apparent power</td>
<td>( S^2 = P^2 + Q^2 )</td>
</tr>
<tr>
<td>Active power</td>
<td>( P = S \cdot \cos \phi )</td>
</tr>
<tr>
<td>Reactive power</td>
<td>( Q = S \cdot \sin \phi )</td>
</tr>
</tbody>
</table>

Note that you get the active power by multiplying the voltage by that part of the current that contributes to the active power, which is the cosine (adjacent/hypotenuse). So, the angle \( \phi \) between \( P \) and \( S \) is the same as that between \( U \) and \( I \).
And you can summarize the performance of your whole system by the Power Factor:

$$\text{PF} = \frac{P}{S} \quad (PF = \cos\phi)$$

The power factor is your primary performance indicator. It is $\cos\phi$, which gives the relation between active and apparent power. A power factor of 0 means that the current is lagging (or leading) the voltage by 90°, while a power factor of 1 means the current and voltage are in phase. So, the smaller the angle $\phi$ the closer is PF to 1, and the more effective your system.

All this scales up – if you have ten motors in parallel you’ll have ten times the current, but if the power factors of all the motors are 0.886, your total power factor will be 0.886 (and, if your utility penalizes you for a power factor below 0.90, then you’ll be paying expensive penalty charges).

**Why worry about reactive power?**

In a motor, the reactive current establishes the alternating magnetic field. The magnetic energy is stored and released each cycle, so the average power is (apart from the relatively small magnetic losses) zero. The supply transformers, however, have to carry the whole motor current – both active and reactive. If you ignore this reactive power, your transformers will overload and overheat. By supplying the current locally, compensating capacitors bring the power factor of your motor back close to $\text{PF}=1$.

Similarly, although you are only using the active power, the utility has to supply both active and reactive. The reactive power is wasted. Worse: the additional reactive current loads your system and causes extra $i^2R$ heat loss. That can reduce equipment lifetimes.
2.2.6 inductive loads – power factor

Going back to our motor, we can see what the canal boat metaphor means for the power to the motor.

We see that the inductive nature of the motor, indicated by $\cos \varphi = 0.866$ (520/600), results in only 520 W out of the 600 W apparent power being transported turning into active power.

The remainder, $600^2 - 520^2 = 300$ var, is reactive power that loads the network but does not contribute to motor torque.

The efficiency of the system, given by the power factor, is 86.6%. The root cause of this inefficiency is the phase shift between current and voltage in non-resistive loads. Shifting the motor current closer in phase to the motor voltage by adding a capacitor to the motor circuit can improve the efficiency of the system. Capacitors are usually considered to generate reactive power, and inductors to consume it. Connecting a capacitor and inductor in parallel cancels out the current.

And, again, apparent power $S^2 = P^2 + Q^2$. 

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>powers with phase shift example 3: motor</td>
<td></td>
</tr>
<tr>
<td>• apparent power: $S = U_{RMS} \cdot I_{RMS}$</td>
<td>$= 600$ VA</td>
</tr>
<tr>
<td>• active power: $P = S \cdot \cos \varphi$</td>
<td>$= 520$ W</td>
</tr>
<tr>
<td>• reactive power: $Q = S \cdot \sin \varphi$</td>
<td>$= 300$ var</td>
</tr>
<tr>
<td>• power factor: $PF = P / S$</td>
<td>$= 0.866$</td>
</tr>
<tr>
<td>• $S$, $P$, $Q$ relation: $S^2 = P^2 + Q^2$</td>
<td>$\varphi$: phase shift between $u(t)$ and $i(t)$ $\cos \varphi = 0.866$</td>
</tr>
</tbody>
</table>
2.3  single-phase power with inductive loads and harmonic distortion

2.3.1  harmonic distortion – going beyond Classical theory

Everything so far has been good Classical theory. Besides the phase angle though, equipment is increasingly producing harmonic distortion, particularly third harmonics, in the current.

Classical theory cannot cope with the effects of harmonics, which was a major reason why IEEE 1459 and the ‘Unified’ theory were developed. So now, we look at how harmonics added to inductive loads affect P, Q, S and power factor.

As a rule of thumb, you can ignore the harmonic content if the % content is lower than the accuracy of your current clamp.

The same motor is now attached to the network, but this time driven by a (noisy) electronic motor drive. We assume the same phase shift as before but in addition 3rd harmonic current is generated by the electronics.

**Instantaneous values**

The instantaneous values are as before: $u(t)$, $i(t)$ and $p(t)$.

**Parameters**

The voltage parameter is $U_{\text{RMS}}$ again, or simply $U$. The current $I$ ($I_{\text{RMS}}$) is now made up of a fundamental component ($I_1$) and the third harmonic ($I_3$). The phase angle $\phi_1$ is here between fundamental current ($I_1$) and the voltage ($U$) – there is only a fundamental voltage, so we just say voltage.
2.3.2 harmonic distortion – simplified by Fourier analysis

Fourier’s theory says that a repetitive signal (top) can be treated as a sum of pure sine waves (bottom).

**Currents**

Once again we’ve chosen $I_{\text{RMS}}$ to be 5 A.

Fourier analysis can separate out the top waveform into a fundamental and a third harmonic component at three times the frequency.

To give the total 5A current in the example above, the fundamental and third harmonic components calculated in Excel turn out to be 4.8 A and 1.4 A. Here, too (because they are RMS currents with a different frequency), the vector sum gives $4.8^2 + 1.4^2 = 5^2$.
2.3.3 harmonic distortion – voltage, current and power

Multiplying the voltage and current waveforms (top) gives the power waveform. And again, because of the motor’s lagging current, we see the power dip below zero (bottom).

Voltages and currents

Because the utility supply is very stable, the voltage consists of the fundamental voltage only, as we’ve seen before.

The RMS current is the full 5A through the load, made up of 4.8 A fundamental and 1.4 A third harmonic.

Powers

We can again use Excel to multiply instantaneous values of voltage and current to find the average power. It turns out to be even lower: 499 W. So, we’ve gone from 600 to 520 to 499 W.

The apparent power that the utility supplies, however, is still 120 V x 5 A = 600 VA.

---

Figure 2.3.3  single-phase power – harmonic distortion – instantaneous and active power

\[ p(t) = u(t) \cdot i(t) \]

\[ P = 499 \text{ W} \]

\[ S = U.I = 600 \text{ VA} \]
2.3.4 harmonic distortion – active, reactive and apparent power

We now repeat our previous trick – and this time we also separate out the harmonic power. So, we separate out the distorted wave of the last diagram into active (green), reactive (red) and harmonic (blue). We’ve already seen that the red reactive power has an average value of zero, and (in this example) this is true too of the blue harmonic power.

Power into the load

The power into the load is now 499 W for the apparent power of 600 VA.

Active and reactive powers

The green fundamental active power is the only wave having a net result: 499 W.

The red fundamental reactive power $Q_1$ is 288 var peak (from Excel, again).

So besides the active power, which is normally the only power that actually does useful work, we have reactive and harmonic powers. Note that, unlike reactive power, harmonic components can cause active power loss because there can be in-phase voltages associated with them.

Classical calculations ignore harmonic influences. So, traditional power quality meters can’t deal with the harmonic component in power – they assume the voltage and current to be sine waves. For many years, with mainly resistive and motor loads, this was fairly well true. Increasing numbers of Switched-Mode Power Supplies in computers and peripheral equipment like printers and copiers introduce harmonic distortion, though. LED lamps are particularly poor because, to save money on compensation circuits, they normally use only a noisy SMPS.
Apparent powers

The utility still sees the full 600 VA apparent power \( S \). Now though we separate it out into the fundamental apparent power \( S_1 \) and the (non-fundamental) apparent power \( S_N \) which collects together all the different harmonic powers (third, fifth, etc.). \( S_N \) is the vector difference between \( S \) and \( S_1 \): \( \sqrt{(600^2 - 576^2)} = 168 \text{ VA} \).

Currents

Once we split out the powers, we can calculate the values of all three currents. The 5 A now separates out into the 4.157 A active, 2.4 A reactive and 1.4 A harmonic. Because it’s calculated using the vector sum, the wasted current is larger than you’d think for a relatively small difference between 5 A and 4.157 A. The network now sees just over 4 A useful current out of 5 A, with about 2.8 A waste current, consisting of 2.4 A reactive and now 1.4 A harmonic current. That is a lot of waste.

<table>
<thead>
<tr>
<th>Harmonic or Non-fundamental?</th>
</tr>
</thead>
<tbody>
<tr>
<td>The terms harmonic (subscript ( H ) as in ( P_H )) and non-fundamental (subscript ( N ) as in ( S_N )) are often mixed up when describing components other than the fundamental. However, they are different.</td>
</tr>
<tr>
<td>A pure harmonic component like ( I_3 ) is part of harmonic current ( I_H ). When talking about the difference from the fundamental component we instead use the term non-fundamental current ( I_N ) (so ( I_N^2 = I^2 - I_1^2 )). Why? Because non-fundamental current includes both harmonic and interharmonic components.</td>
</tr>
<tr>
<td>For a 60Hz fundamental component, for example, a 120Hz component is a harmonic but a 100Hz component is an interharmonic. If the current contains 60Hz, 100Hz and 120Hz components, the 120Hz component is harmonic, the 100Hz component is interharmonic, and the 100Hz and 120Hz components together are the non-fundamental components.</td>
</tr>
<tr>
<td>In this document we use subscript ( H ) for harmonic and subscript ( N ) for non-fundamental. To avoid confusion with components of the neutral conductor in three-phase systems we use the subscript ( n ) (lower case) for neutral currents, voltages and powers – in line with IEEE 1459.</td>
</tr>
</tbody>
</table>
2.3.5 harmonic distortion – where does it come from?

Applying a pure sine-wave voltage to a linear load produces a pure sine-wave current. For linear loads like heaters, incandescent lights and motors, the value of current is simply proportional to the voltage. Linear loads do not change the shape of the current waveform, although they may change the phase relationship between voltage and current (linear loads often refer to resistive loads, but may actually be resistive, inductive or capacitive).

Applying a sine-wave voltage to a non-linear load produces a current that contains harmonic components (the value of a non-linear load changes with the voltage applied to it). The most widespread example of a non-linear load is the rectifier circuit. This is widely used in switched-mode power supplies, variable-speed motor drives, LED and compact fluorescent lights and many other electronic devices.

The AC input voltage $U_{ac}$ in the single-phase bridge rectifier circuit (top right) is fed to a diode bridge D by a series impedance $R_s$. The output charges the capacitor C which supplies the DC voltage $U_{dc}$ to $R_{load}$.

**The voltage and current waveforms**

In the top graph the solid red line shows the instantaneous AC voltage $u(t)$ applied to the rectifier bridge. The dotted red line shows the rectified part of $u(t)$, written as $|u(t)|$. The blue line shows the DC voltage $U_C(t)$ across the capacitor. Notice that this voltage is not constant but has a ripple on it.

At the left of the graph the voltage across the capacitor is about 120 V and decreasing because $R_{LOAD}$ is discharging the capacitor. While $u(t)$ is smaller than $U_C(t)$, the diode bridge will not conduct because D1 is reverse biased and no current will flow. When the increasing $u(t)$ equals the decreasing $U_C(t)$, at a phase angle of about 30°, the diodes D1 and D2 become forward biased and current $i(t)$ starts to flow (bottom graph).

**figure 2.3.5 single-phase power – harmonic distortion – bridge rectifier**

- $u(t) = 120V/60Hz$ sine
- $U_C(t) = 100–170V$ saw
- $i(t) \approx 4.05A/60Hz$ pulse
- $I_1 \approx 3.90A/60Hz$ sine
- $I_3 \approx 1.06A/180Hz$ sine
- $I_5 \approx 0.11A/300Hz$ sine
The value of current is limited by the series impedance $R_s$. From this point on the current follows the voltage as if the load were linear. The capacitor is charged by this current and $U_C(t)$ increases with $u(t)$ until $u(t)$ reaches the top of the sine wave and starts to decrease again (at a phase angle of 90°). The diode bridge becomes reversed biased again and the current $i(t)$ stops flowing. The capacitor is no longer being charged and the voltage $U_C(t)$ starts to decrease again as it discharges into the load $R_{load}$.

The procedure repeats for the negative half cycle of $u(t)$ but now with the rectified (dotted) part of $u(t)$; D1 and D2 work on the positive half of $u(t)$ and D3 and D4 of the negative half. The current $i(t)$ has the same form as in the first half of the cycle, but now is negative. The current $i(t)$ does not look like a sine wave but more like an impulse signal.

**Significant 3rd and 5th harmonics**

This is typical for these types of rectifier circuits. Fourier’s theory says that all periodic signals can be decomposed into harmonic sine waves, and if we decompose this current signal we find large 3rd and 5th harmonic components. For an actual (less simplified) circuit, if the RMS value of the total current is 100% then the fundamental (60Hz) RMS component is 96%, the 3rd harmonic (180Hz) RMS component is 26% and the higher uneven harmonic components (300 Hz and higher) are 10% of the total RMS. Remember to get the total RMS you have to add the squares of the components.

As the use of electronic power devices with rectifiers has increased dramatically over the last decades, the pollution of the electrical grid with higher harmonics has also increased. These harmonics create losses in power lines and transformers, mostly in the form of heat. That is why international standards like IEC and IEEE specify the maximum harmonics that may be caused by larger pieces of equipment. However, as the number of small electronic devices like cheap LED lights increases, the harmonic pollution of the mains will remain a problem.

Countermeasures include more sophisticated rectifier circuits and harmonic filters in front of each device, but these increase the costs of individual products. With many such noisy devices, you will need separate compensation circuits.
harmonics: $S^2 \neq P^2 + Q^2$ why?

**2.3.6 harmonic distortion – why is $S^2$ no longer just $P^2 + Q^2$?**

We now can’t just say $S^2 = P^2 + Q^2$ because we have to include $S_n^2$, the non-fundamental apparent power squared. So $S^2 = P^2 + Q^2 + S_n^2$.

Continuing with the canal boat analogy, adding harmonics is like pulling the canal boat through choppy water. The waves cause a third force, perpendicular to the other two, which represents the non-fundamental apparent power.
2.3.7 harmonic distortion – a summary of calculations

This page summarizes the calculations on the E-device so far. You can check through the figures afterwards, we’ll here just concentrate on the relationships.

Active, reactive and apparent powers

The active power $P$ is here the same as the fundamental active power ($P_1 = S_1 \cdot \cos \phi_1$). That is because we defined the harmonic voltage to be zero, so there is no harmonic contribution $P_H$ in this example. In general $P = P_1 + P_H$ where $P_H$ is the average value of $p_H(t)$. In this example $P_H = 0$, so $P = P_1$.

The corresponding fundamental reactive power ($Q_1 = S_1 \cdot \sin \phi_1$) is the largest reactive power component, and the only important one.

The apparent power $S$ supplied by the utility splits into the fundamental apparent power $S_1$, and the non-fundamental apparent power $S_N$ (which collects together all the harmonic powers)

Active, reactive and harmonic currents

The Unified method allows us to find the active, reactive and non-fundamental currents and voltages, which IEEE 1459 does not define explicitly. Our example only shows the current components, as we assume the voltage to be purely sinusoidal.

Note that, while $P$ and $Q$ can be negative, so pushing power back into the mains, $S$ is always positive.
### 2.3.8 harmonic distortion – performance parameters

Performance parameters represent the efficiency of the system. There are several different parameters, depending on exactly what you want to measure in the circuit.

The **power factor** is the total active power over the total apparent power: useful because it shows what the system is providing. The closer to 1, the more efficient is your system.

Similarly, the **fundamental power factor** (often called the displacement power factor in the US) is the fundamental active power over the fundamental apparent power. Europeans here use $\cos \phi$, which gives basically the same value, but it is sometimes easier to measure the individual power components. In most practical cases, only the fundamental component of the power does useful work. The non-fundamental active power will sometimes be useful in a heater, but otherwise not.

The **efficiency factor** (defined in the Unified method) is the useful (i.e. fundamental) part of the real power divided by everything you have to transport.

The **harmonic pollution** is the non-fundamental apparent power over the fundamental apparent power. It is a quick estimate of the amount of harmonic pollution in the system. Smaller is better!

**Adding some numbers**

Comparing power factor and fundamental (displacement) power factor shows the harmonic content.

The power factor and efficiency factor are here the same because there is here no harmonic voltage in our example, and so no harmonic active power.

While the power factors should ideally be 1, the harmonic pollution should be 0 (which is itself very difficult).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Formula</th>
<th>Example Calculation</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>Power factor</td>
<td>$PF = \frac{P}{S}$</td>
<td>$PF = \frac{499}{600} = 0.832$</td>
<td></td>
</tr>
<tr>
<td>Fundamental power factor</td>
<td>$DPF = \frac{P_1}{S_1}$</td>
<td>$DPF = \frac{499}{576} = 0.866$</td>
<td></td>
</tr>
<tr>
<td>Efficiency factor</td>
<td>$EF = \frac{P_1}{S}$</td>
<td>$EF = \frac{499}{600} = 0.832$</td>
<td></td>
</tr>
<tr>
<td>Harmonic pollution</td>
<td>$HP = \frac{S_N}{S_1}$</td>
<td>$HP = \frac{168}{576} = 0.292$</td>
<td></td>
</tr>
</tbody>
</table>
Which performance measure you use depends on what you are looking for. If your system has no harmonics then you’d just use the power factor. With harmonics floating round your system, you’d be more interested in the fundamental (displacement) power factor or the efficiency factor. The efficiency factor is actually often the most accurate measure of system performance since it shows the useful power against the power taken from the utility.
### AC example summary

<table>
<thead>
<tr>
<th></th>
<th>lamp</th>
<th>motor</th>
<th>e-device</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>voltage:</strong></td>
<td>full U</td>
<td>120 V</td>
<td>120 V</td>
</tr>
<tr>
<td><strong>current:</strong></td>
<td>full I</td>
<td>5 A</td>
<td>5 A</td>
</tr>
<tr>
<td></td>
<td>fund. I₁</td>
<td>5 A</td>
<td>5 A (-30)</td>
</tr>
<tr>
<td></td>
<td>fund. I₃</td>
<td>0 A</td>
<td>0 A</td>
</tr>
<tr>
<td><strong>active power:</strong></td>
<td>full P</td>
<td>600 W</td>
<td>520 W</td>
</tr>
<tr>
<td></td>
<td>fund. P₁</td>
<td>600 W</td>
<td>520 W</td>
</tr>
<tr>
<td><strong>reactive power:</strong></td>
<td>fund. Q₁</td>
<td>0 var</td>
<td>300 var</td>
</tr>
<tr>
<td><strong>apparent power:</strong></td>
<td>full S</td>
<td>600 VA</td>
<td>600 VA</td>
</tr>
<tr>
<td></td>
<td>fund. S₁</td>
<td>600 VA</td>
<td>600 VA</td>
</tr>
<tr>
<td></td>
<td>fund. Sₙ</td>
<td>0 VA</td>
<td>0 VA</td>
</tr>
<tr>
<td><strong>power factor:</strong></td>
<td>full PF</td>
<td>1.000</td>
<td>0.867</td>
</tr>
<tr>
<td></td>
<td>fund. DPF</td>
<td>1.000</td>
<td>0.866</td>
</tr>
</tbody>
</table>

#### Figure 2.4.1  single-phase power – summary – example parameter values compared

### 2.4 single-phase power summary

#### 2.4.1 summary – example parameter values

This slide summarizes the key AC parameters. We can see that many are the same for lamp, motor and electronic device.

**Voltages and currents**

We have engineered the examples so that the full RMS voltages and currents appearing across the load are the same, for easy comparison of the three examples.

The fundamental current is the same (but delayed by the phase angle with inductive loads) until you get harmonic current in electronic devices.

**Powers**

The **full active power** (the average of \( p(t) = u(t).i(t) \)) drops when we have a phase shift between voltage and current, and further if there are harmonics.

The **fundamental active power** drops in the same way because we have assumed no harmonic voltages, hence no harmonic active power. (We have assumed in all the examples that the voltage is a sine wave. Although the Fluke 430 Series II Power Analyzer will calculate the results with harmonic voltages – at the end of a long transmission line, for example – we have left these out from the discussion. The principle is the same, but it would introduce more complicated calculations.)

**Reactive power** appears as soon as you have a non-resistive load, producing a phase shift (note that the reactive power drops in electronic devices because these decrease the fundamental current).
The full apparent power (fundamental plus harmonics) is always the same: RMS voltage x RMS current. This is the value the utility sees and so charges you for. The fundamental apparent power is only different if there are harmonics. So, if there is a difference between full apparent power and fundamental apparent power, you have harmonic currents in electronic devices somewhere.

Power factors

Starting from 1, the power factor drops because of the reactive power, and drops more because of the harmonic power. The fundamental power factor DPF drops the same amount because of the reactive power, then drops no further because DPF excludes the harmonic influence.
### 2.4.2 summary – single-phase power formulae

This diagram summarizes the key formulae.

#### Voltages and currents

To add RMS values, you use vector addition.

#### Active and reactive powers

To find the full active power, you need to go back and take the instantaneous voltage over time multiplied by the instantaneous current through the load over time – calculated by digital power meters by multiplying the sampled voltages and currents. With close enough samples this is accurate, although to be mathematically correct you would have to integrate over a period. It is the only good way to deal with a signal that you don’t know about.

If you know that your signal is a sine wave, as with the fundamental component, then the fundamental active power is simply the fundamental apparent power $S_1$ multiplied by $\cos\varphi_1$ (the angle between fundamental voltage and current). The active harmonic power is the full active power minus the fundamental active power.

The fundamental reactive power of a sine wave is similarly $S_1$ multiplied by $\sin\varphi_1$. Be careful using this relation for calculating the full powers $P$ and $Q$ though, since this is where the Classical method can go wrong. It only works for sine waves – so it is fine for calculating $P_1$ and $Q_1$ but only for $P$ and $Q$ if there are no harmonics! If the harmonic component is small then the error will only be small. But the larger the harmonic component, the less accurate will be your result.

---

**Figure 2.4.2 single-phase power – summary – formula summary**

<table>
<thead>
<tr>
<th>Voltage:</th>
<th>Full</th>
<th>$U^2 = U_1^2 + U_H^2$</th>
<th>$V_{(RMS)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Current:</td>
<td>Full</td>
<td>$I^2 = I_1^2 + I_H^2$</td>
<td>$A_{(RMS)}$</td>
</tr>
<tr>
<td></td>
<td>Fund.</td>
<td>$I_1^2 = I_a^2 + I_r^2$</td>
<td>$A_{(RMS)}$</td>
</tr>
<tr>
<td>Active power:</td>
<td>Full</td>
<td>$P = \text{avg}[u(t).i(t)]$</td>
<td>$W$</td>
</tr>
<tr>
<td></td>
<td>Fund.</td>
<td>$P_1 = S_1 \cdot \cos(\varphi_1)$</td>
<td>$W$</td>
</tr>
<tr>
<td></td>
<td>Harm.</td>
<td>$P_H = P - P_1$</td>
<td>$W$</td>
</tr>
<tr>
<td>Reactive power:</td>
<td>Fund.</td>
<td>$Q_1 = S_1 \cdot \sin(\varphi_1)$</td>
<td>$\text{Var}$</td>
</tr>
<tr>
<td>Apparent power:</td>
<td>Full</td>
<td>$S = U \cdot I$</td>
<td>$\text{VA}$</td>
</tr>
<tr>
<td></td>
<td>Fund.</td>
<td>$S_1 = U_1 \cdot I_1$</td>
<td>$\text{VA}$</td>
</tr>
<tr>
<td></td>
<td>Harm.</td>
<td>$S_H^2 = S^2 - S_1^2$</td>
<td>$\text{VA}$</td>
</tr>
</tbody>
</table>
Apparent powers

The **apparent power** seen by the utility is simply voltage times current measured at the distribution board. Similarly the **fundamental apparent power** is fundamental voltage times fundamental current. To find the **total harmonic apparent power**, you take a vector subtraction of the fundamental component from the full apparent power. So you don’t calculate the non-fundamental (apparent) power itself, you calculate the total and then subtract the fundamental. The non-fundamental apparent power is what is left over.

Note that you don’t actually need the Fourier transform for power analysis, you just need some way of finding the full power and some way of finding the fundamental power. In the analog world, you could do that with heavy filtering at 60 Hz. Note too, that $S_N$ doesn’t distinguish between third, fifth, seventh (etc.) harmonics – all that are important are the full and fundamental harmonic powers.
Here, we summarize the key power and performance parameters. Both IEEE 1459 and the Unified method distinguish between apparent, active and reactive power, and you can fill this table with the various values.

Full values include all signal components, which separate out into fundamental values and non-fundamental values (everything except the fundamental).

**Apparent power**

The full apparent power $S$ (always in Volt Amps) is the fundamental apparent power $S_1$ plus the harmonic (non-fundamental) apparent power $S_N$.

**Active power**

Similarly for the active powers: the full active power $P$ is the fundamental active power $P_1$ plus the harmonic active power $P_H$. You can calculate each harmonic separately ($P_H = U_3 \times I_3 \times \cos \phi_3 + \ldots$) for as many harmonics as appear, but it is easier to find $P_H = P - P_1$. Whether this power is wasted or not depends on the load – once again if the load is a heater then it will give you useful power. In motors and most electronic equipment, the power is simply converted into heat, and so wasted.

**Reactive power**

Because the fundamental component of the power dominates energy transport, we consider only the fundamental reactive power (always in volts amps reactive, var). It is not useful to split out reactive powers from the individual harmonic components.

**Performance parameters**

There are performance parameters for full powers (power factor), fundamental powers (displacement power factor) and harmonic powers (harmonic pollution).

---

**Figure 2.4.3 single-phase power – summary – parameter summary**

<table>
<thead>
<tr>
<th>POWER</th>
<th>full</th>
<th>fund.</th>
<th>non fund.</th>
</tr>
</thead>
<tbody>
<tr>
<td>apparent</td>
<td>$S$ (VA)</td>
<td>$S_1$ (VA)</td>
<td>$S_N$ (VA)</td>
</tr>
<tr>
<td>active</td>
<td>$P$ (W)</td>
<td>$P_1$ (W)</td>
<td>$P_H$ (W)</td>
</tr>
<tr>
<td>reactive</td>
<td>- - -</td>
<td>$Q_1$ (var)</td>
<td>- - -</td>
</tr>
<tr>
<td>perf. param.</td>
<td>$PF = P / S$</td>
<td>$DPF = P_1 / S_1$</td>
<td>$HP = S_N / S_1$</td>
</tr>
</tbody>
</table>
three-phase power

In this section we discuss:

3 three-phase AC power
  3.1 introduction
  3.2 balanced sine-wave systems
  3.3 wye and delta systems
  3.4 unbalanced sine-wave systems
  3.5 symmetrical components
  3.6 unbalance sine-wave examples
  3.7 combined power (unbalance and harmonics)
  3.8 neutral currents
  3.9 power summaries

Figure 3.1.1 three-phase power – introduction

3 three-phase AC power

3.1 introduction to three-phase power

3.1.1 introduction – general

Larger pieces of equipment like the large compressors that run chiller plants, fans on air handling units, and motors on process machines run on three-phase power. Ideally, these consume the power equally from each of the three phases. However, that often doesn’t happen. Distributing equipment across a facility can itself cause the currents at the distribution board to be unbalanced, resulting in energy loss and possible voltage unbalance.

This variation can cause instability in three-phase equipment, and if for example the voltages on a three-phase electric motor are slightly different the motor will rotate with some imbalance. That can cause motor vibration which reduces both efficiency and lifetime. Unbalance voltages can also cause malfunctions in single-phase loads that are connected to a phase whose voltage is lower or (particularly to be avoided) higher than the load’s rated voltage.

While single-phase circuits can be measured using a clamp meter or multimeter, three-phase systems need a power quality meter. The latest 430 Series II meters can detect unbalance, which can be reduced by rearranging the loads and/or installing passive or active unbalance compensation equipment.

We now present basic three-phase power and introduce three-phase vector (phasor) diagrams. Because three-phase resistive loads are straightforward, we go straight into inductive loads (which also incorporate a resistive component). If all three phases are balanced, they all act like three independent single-phase loads. Any differences between the loads, though, and you get unbalance – we describe how to deal with this. Finally we introduce harmonics again, and give some examples and an overview.
### three-phase symbols & colors

- **phase identification:**  
  - A, B, C, N, Gnd  
  - L1, L2, L3, N, Gnd

- **phase color coding:**  
  - A/L1  
  - B/L2  
  - C/L3  
  - N  
  - Gnd

**US:**

- ![US phase colors](image)

**EU:**

- ![EU phase colors](image)

**UK:**

- ![UK phase colors](image)

**HD 308 S2:**

- ![HD 308 S2 phase colors](image)

---

**Figure 3.1.2** three-phase power – introduction – phase symbols and color coding

### 3.1.2 introduction – symbols and colours

The phases are shown differently in different countries (ABC used to be called RST, but that no longer appears in official documents).

**Important note:** the red, green and blue phase colours in this document are again just for demonstration and do not follow any standard. It is *not* the same as the colour coding in individual countries, nor in the HD 308 S2 cable standard so ‘don’t try this at home’.
### 3.2 three-phase balanced sine-wave systems

#### 3.2.1 balanced sine-wave – basic system

This is the diagram of a basic three-phase power system with inductive (and resistive) loads. You can think of it as three single-phase systems connected to a neutral line (so four wires). Indeed for domestic supplies, utilities normally connect their three-phase supplies to three (single-phase) houses: one phase to each in turn – on average the loads will balance out.

We’ve labelled the voltages and currents A, B, C and N according to the American convention that is followed in IEEE 1459 (the European equivalents are L1/L2/L3/N). The red, green and blue phase colours here are just for demonstration and do not follow any standard.

Now we have a three-phase AC load with each phase taking 600 VA (60 Hz 120 V again), so 1800 VA in total.

**Instantaneous values**

There are four voltages, currents and powers – with subscripts A, B, C and N.

**Parameter values**

Besides the four voltages and currents there are four powers (subscripts A, B, C and N again) and a total power P. As we shall see, the power for the total load is not always the sum of the A, B, C powers.

We have $P = P_A + P_B + P_C$ and $Q = Q_A + Q_B + Q_C$ but in general $S \neq S_A + S_B + S_C$

In general, the neutral power is treated separately. It is generated by the neutral current into the impedance of the neutral conductor, so does not add to the useful transfer of energy and is considered as a loss component. It depends whether you are looking only at the load or at the total system as to whether you take it into account or not. Seen from the utility side, however, neutral power is a waste of energy.
3.2.2 balanced sine-wave – voltage and current waveforms

We have three voltages and three currents. Ideally each voltage lags the previous one by 120° (look at the zero crossings). We take $u_A(t)$ as the reference at 0°, so $u_B(t)$ is at -120° and $u_C(t)$ at -240°.

The motor again introduces its own 30° phase shift between voltage and current. So compared to $u_A(t)$ again the phase angle of $i_A(t)$ is -30°, $i_B(t)$ is -150° and $i_C(t)$ is -270°.

**Instantaneous values**

We represent (for example) the instantaneous value of voltage $u_A(t)$ by the parameters 120 V RMS and phase angle $\angle 0°$.

Note that the sum of each of the voltages (and each of the currents) at the star point is always zero. With each of the three sine waves being delayed by exactly 120 degrees, the voltage across two of the conductors always balances the third. A peaks first, followed by B followed by C.
3.2.3 balanced sine-wave – vector diagram

To avoid having to draw hundreds of sine waves, we can show them in a vector (phasor) diagram – just another way to represent the same information. This vector diagram shows just the fundamental values, with the lengths of the lines representing the RMS values, and their heights above the origin showing their instantaneous values. In this example, all voltages are the same, and all currents are the same.

Parameter values

This three-phase vector diagram shows $U_{1A}$ as reference again at 0° (the phase waveforms correspondingly showed the value of $U_{1A}$ as 0 at 0°). Once again the B and C phase voltages lag by 120° and 240°, and the A, B and C phase currents lag by 30°, 150° and 270°.

The whole thing turns at 60 times/sec (shown by the rotating arrow), but this doesn’t really add any information so we freeze it with $U_{1A}$ on the horizontal axis. By convention the diagram shows the vectors turning counter-clockwise so here too A passes first then B then C, with the angle continually increasing over time. After 360°, the action repeats.

The vector diagram can only be drawn for one frequency component at a time. Mostly this is the fundamental component, but it could also be drawn for each harmonic component.
3.2.4 balanced sine-wave – positive sequence

We define the waveforms going from A to B to C as being positive sequence. Phasors rotate counterclockwise. First is A, drawn at the 0 degrees position, then follows B and C.
3.2.5 balanced sine-wave – negative sequence

Swap the B and C connections and you get a negative sequence – rather than ABC the motor rotates ACB (so in reverse). The phasors still turn counter clockwise. A is still first, but than follows C and B comes last.

![Diagram of three-phase power – balanced – negative-sequence vector diagram](image)

**positive sequence**
A – B – C
(motor runs forward)

swapping B – C phase:
**negative sequence**
A – C – B
(motor runs in reverse)
positive & negative sequence waveform

positive sequence:
\[ u_A(t) : 120V \angle 0^\circ \]
\[ u_B(t) : 120V \angle -120^\circ \]
\[ u_C(t) : 120V \angle -240^\circ \]

negative sequence:
\[ u_A(t) : 120V \angle 0^\circ \]
\[ u_C(t) : 120V \angle -120^\circ \]
\[ u_B(t) : 120V \angle -240^\circ \]

Figure 3.2.6  three-phase power – balanced – positive- and negative-sequence waveform

3.2.6  balanced sine-wave – positive- and negative-sequence waveforms

Once again, positive sequence gives first A then B then C, while negative sequence gives A then C then B.
3.2.7 balanced sine-wave – phase and system-level parameters

Look again at the three-phase motor.

We first look at a balanced system – a source with balanced (equal) voltages driving a balanced load resulting in balanced currents.

**Instantaneous values**

The instantaneous sum of all three voltages is always zero and there is no neutral voltage, with the same conditions for the currents.

**Parameters**

The RMS values are similarly all equal, and there is again no neutral voltage or current.
3.2.8 balanced sine-wave – phase and system-level powers

With a three-phase balanced system, we can treat each individual phase as a separate single-phase system. So, we can refer to the phases as X instead of A, B, and C.

Phases

The equations for power are equivalent to those for the single phases with motor attached.

System

At the system level, the apparent powers \( S_a \) and \( S_{a1} \) (with lower-case a) are found by adding the values for the three phases arithmetically. The subscript “a” stands for (Classical) arithmetic, which means simply adding. This is again all good Classical theory, but is only true for a balanced system with sine waves having no harmonics (and is therefore the reason for the Classical theory’s limitations). This is however what your utility would like to see – anything outside this is unwanted and you pay for it.

With Buchholz’s definition of three-phase reactive and apparent power, we’ve entered the 20th Century. From this point in the tutorial, we start to go beyond Classical power theory. Anywhere you have an unbalanced system, or where you have harmonics, you can’t calculate apparent power by simply adding the power in the three phases. The question, though, is how you calculate the apparent power – the power that appears to be taken from the utility.

Because the different theories differ in how they calculate \( S \), the following subscripts are used: \( S_a = \) arithmetic \( S \) (Classical), \( S_e = \) effective \( S \) (IEEE 1459), and \( S \) (without subscript) = Buchholz’s model \( S \).

Buchholz’s definition is important because it has been adopted by many power specialists, and also by the Unified approach. It defines apparent power from the voltage and current waveforms, basically calculating the maximum active power that can be transmitted to a load for a given voltage value).
The same subscripts are used for the fundamental apparent powers $S_{a1}$, $S_{e1}$ and $S_1$.

Remember that you can again always simply sum the active powers $P$, fundamental active powers $P_1$ and fundamental reactive powers $Q_1$ because they are along the same axis.

$S_a$ = arithmetic apparent power (Classical)
$S_e$ = effective apparent power (IEEE 1459)
$S$ = Buchholz, DIN and Unified apparent power

You can simply add the active and fundamental active powers $P$ and $P_1$ and the fundamental reactive power $Q_1$. You can only add the apparent powers if there are no harmonics and no unbalance.
Figure 3.2.9  three-phase power – balanced – calculation results overview

3.2.9  balanced sine-wave – voltage, current and power overview

This list shows all the results of the calculations for this balanced three–phase motor without harmonics. So, Classical power theory in a single sheet.

**Voltages and currents**

The voltages and currents are all straightforward – balanced across the phases with nothing going into the neutral.

**Powers**

Similar with the powers. Each phase acts as we would expect from a single-phase motor, and the system powers are three times those of the individual phases. The system power factors are the same as the (identical) power factors across the phases. However, they are not derived from the individual phases, but by dividing the system powers \( S_a/P \) and \( S_{a1}/P_1 \).
3.3 wye and delta configuration

3.3.1 wye and delta – basic circuits

Two configurations of three-phase power are popular: 4-wire WYE (Y) systems and 3-wire delta (Δ) systems. The difference lies in how the loads are connected, and in that Y has neutral and Δ does not. (The symbols reflect the configuration: the branches of the symbols ‘Y’ and ‘Δ’ hold the loads – the neutral comes out of the middle of the Y to give the fourth wire, while the three-wire connections come from each point of the Δ.)

The four-wire Y has line voltages (to neutral) and line currents. This is the circuit we have been studying. The three-wire Δ also has line currents, but line-line voltages $U_{AB}$, $U_{CA}$ and $U_{BC}$ instead of line voltages.

If you look from the outside into the circuit, you can’t see whether it is wired Y or Δ, the only thing you can see is whether it has a neutral or not. For loads with the same impedances, the currents $I_a$, $I_b$, and $I_c$ are the same for both Y and Δ. Similarly, you can’t tell from the outside which part of $I_c$ flows through loads CA or BC (the line-line currents), unless you know the exact impedances of the loads, which is normally not true! Only with a perfectly balanced load can you calculate the relations between $I_a$, $I_b$, $I_c$ and $I_{AB}$, $I_{BC}$ and $I_{CA}$.

We’ll now look at the differences between wye and delta systems.
3.3.2 wye and delta – vector diagram

The diagram shows the line voltages (for Y) the line-line voltages (for Δ), and the line currents (for both Y and Δ).

For Δ there are no individual load (line-line) currents, which complicates things somewhat. You know the line-line voltages and line currents but you can’t directly relate them to each other. Note that the phase relation between $U_{AB}$ and $I_A$ is not same 30° we had before (although $U_{AB}$ in a Delta system does not go through the origin, it is the angle between $U_{AB}$ and $I_A$ that is important: in the diagram it is the angle at which the $U_{AB}$ vector crosses the $I_A$ vector).
3.3.3 wye and delta – a summary of differences

Four-wire Y system

For wye there are line voltages and line currents. There can be a neutral voltage and neutral current (although ideally these would be zero). We know the line powers, and we can find the system powers simply from the line powers.

Three-wire Δ system

Generators are often delta connected to distribution transformers, and these systems are somewhat more difficult to analyze than wye systems. With delta systems we know the line-line voltages and line currents. We don’t know the line-line currents, though. There is no neutral so no neutral voltage or current. We don’t know line-line powers, and you can’t find the system power from the line powers any more. Instead, you have to measure it using the two-wattmeter method.

We can find the system power for a Y configuration directly from the phase powers but need a way to find it for a Δ system.
3.3.4  wye and delta – power in delta systems with two-wattmeter method

Blondel’s theorem of 1893 is that – in any system of electrical conductors – you need one fewer power meter than you have wires: so for a three-wire Δ system you need two power meters. This is the general case for the Aron circuit (the first accurate Watt-hour meter, introduced by Aron in 1888) shown here. Although harmonics and unbalance were not a problem at the time, the circuit finds them too because it measures instantaneous powers.

The Aron circuit uses one wire (B) as a reference for two wattmeters (the 430 Series II actually incorporates two wattmeters to allow you to find the system powers in a Δ system). The upper one uses the voltage between A and B and the current into A, and the lower one uses the voltage between B and C and the current into C.

Although not a strict mathematical proof, the following illustrates how we get the result.

Powers

We’ve said that the total instantaneous power is the sum of the powers in the phases. So we can substitute the instantaneous voltages and currents (which are again functions of time).

We know some things about this circuit. The sum of the currents is zero, so we can substitute the current into B into the expression for \( p(t) \) and rearrange the terms.

Since \( (u_A - u_B) \) at any time is \( u_{AB} \) and \( (u_C - u_B) \) at any time is \( u_{CB} \), we can find \( p(t) \) and so the average power \( P \) from the two wattmeters (strictly, again, it is actually \( 1/t \) times the integral).
three-phase power – wye and delta – delta two-wattmeter method calculations

3.3.5 wye and delta – delta powers and power factor

Active, reactive and apparent powers

From the last diagram, the active power (which does the work) is \( P = \text{avg}[u_{AB}(t) \cdot i_A(t) + u_{CB}(t) \cdot i_C(t)] \)

For the fundamental active power \( P_1 \), we can use the RMS values of fundamental voltages and currents with \( \cos \phi \) (the angle between them). Note that you can’t separate out the \( P_{AB} \) and \( P_{CB} \) parts of the equation, they can only be used as a pair: together they form the fundamental active power.

The same holds for fundamental reactive power \( Q_1 \) but using \( \sin \phi \).

For simplicity, we start with the arithmetic formulae for apparent power, so \( S_a \) and \( S_{a1} \). This is only true with no unbalance and no harmonics, but is the most intuitive approach and, when there are indeed no unbalance and no harmonics, it does the job! The subscript \( a \) (lowercase) again stands for arithmetic.

All the powers shown here (\( P, Q \) and \( S \)) for delta connections are system level powers, the result of combined phase powers. Phase powers are given subscript \( A, B, C, N \), or in general \( X \) (all uppercase subscripts). It is all somewhat confusing, but this is how it is done in literature.

Power factors

We can calculate the power factor at system level because it needs only knowledge of system-level powers \( P \) and \( S_a \). We can similarly calculate the DPF at system level because we know \( P_1 \) and \( S_{a1} \). However, we can no longer link this number with an angle between a physical voltage and current.

So, we can now find the power factors of both \( Y \) and \( \Delta \) systems.
Remember that the currents flowing into a Δ system are still Y currents, but without a neutral. Note, too, that if the Δ loads are not equal, the phase currents and the Δ powers will be unbalanced. Although there is no resulting neutral current, a large unbalance current can run round the delta, invisible from the outside but still leading to unbalance power.

Unbalance is most easily demonstrated in Y systems so we will from now on again mainly consider them, pointing out the differences with delta systems when they are important. The procedures for calculating unbalance are basically the same for Y and Δ, but the differences lie in the equations used.

If your buildings or processes are well behaved, with no large inductive loads and only a few computers and peripherals, then you’ve pretty well reached the end of this tutorial. You will have balanced loads with very little current into neutral, and your apparent power will be given by \( S^2 = P^2 + Q^2 \). You can still make savings by improving the efficiency of energy consuming equipment – reducing the active power at source – but your power quality losses will generally be small.

If you are likely to have unbalance, distortion or both in your network, though, read on.
3.4 three-phase unbalanced sine-wave systems

3.4.1 unbalanced sine-wave – unbalanced currents from inductive loads

We’ve seen what happens with a balanced wye system having inductive loads: the voltages and currents are all equal across the phases. The whole simply acts as three independent single-phase systems with nothing going into the neutral. Now we start to look at unbalance. Unbalance effects can be especially severe when equipment is improperly installed, or fails, or where photovoltaic solar power systems are connected to a single phase.

You can distinguish between voltage unbalance (source unbalance) and current unbalance (load unbalance). Voltage unbalance is mostly caused by current unbalance in the transmission line, and current unbalance is in turn mostly caused by load unbalance. In general, the unbalanced load is the root cause of the problem because the current unbalance is normally far greater than voltage unbalance.

Balanced voltage

So, we’ll keep things simple and continue to assume a balanced voltage \(U\) with phases all having the same voltage and separated by 120°. To this we connect unbalanced loads.

Unbalanced current

The currents are now unbalanced: the magnitudes can be different and the phase angles can be different. Note that, like harmonic power, unbalance components can cause active power loss because there can be voltages associated with them.

You can compare unbalance to people carrying a medieval sedan chair. If all your carriers are the same height and they all step in synchronism, then the person in the compartment will travel in style. Carriers all of different heights will make for an unbalanced and bumpy ride.
3.4.2 unbalanced sine-wave – apparent power vector diagram

With current unbalance, we can no longer say that $S$ is simply the arithmetic sum of the different phases.

This simplified vector diagram shows the voltages and phases of the previous diagram. The $P$ values are drawn along the x axis while the $Q$ values are drawn along the y axis. $S_A$, $S_B$ and $S_C$ again have not only different lengths but can have different directions. The vector sum is smaller than the Classical method of the arithmetic sum (projected below). Note that this is a simplified vector diagram, and does not show the effect of harmonics and unbalance. It just shows that simply adding is not the right way.

This suggests that taking the vector sum is the right way to calculate $S$. Generally it leads to better results than using the arithmetic sum. However in practice calculating the apparent power with unbalance and harmonics is more complicated. The correct approach is still the subject of fierce academic debate: the harmonics and unbalance both add to the apparent power, but the question is how.

In general the actual values of system apparent power $S$ and fundamental apparent power $S_1$ are larger than the Classical arithmetic or vectorial calculations because under conditions of unbalance and (not shown on the diagram) harmonics, the utility has to supply more power than without these components. The calculations for $S$ and the fundamental apparent power $S_1$ lie at the heart of the differences between the Classical, Unified and IEEE 1459 approaches – which we now discuss.

Remember you can always add $P$'s and you can always add $Q$'s because they point in the same direction, either horizontal or vertical. Once again, $S$ is special. You can only add $S$ if you know there is very little harmonic distortion and unbalance. Otherwise, you have to go through the more complicated calculations given in IEEE or Unified theory.
apparent power in three-phase systems

how can we measure apparent power in a three-phase system?

- **Classical**:
  - arithmetic: Use the mathematical sum of the phase powers. (this calculation gives incorrect results if harmonics or unbalance are present in the system)
  - vectorial: Use the vector sum of the phase powers. (this calculation gives some better results for unbalance but still ignores harmonics)

- **IEEE 1459**:
  Construct a virtual balanced replacement system with the same external losses as the original unbalanced system.
  This system is called: Effective System
  (this calculation gives large values in the presence of large neutral currents)

- **Unified Power Measurement (UPM)**:
  Use the real system and calculate apparent power based on a transformation of the instantaneous voltages and currents in the system.
  (this calculation gives more realistic values in the presence of large neutral currents)

Figure 3.4.3  three-phase power – unbalance – apparent power calculation methods

3.4.3  unbalanced sine-wave – apparent power calculation methods

You can simply add the active powers of the different phases, and you can simply add the reactive powers of the different phases. But how do you measure apparent power in an unbalanced system? That is the interesting one, because it defines the system load: the load on the network.

There are different methods of calculating apparent power, leading to different answers, because the definition of apparent power is ambiguous in the presence of unbalance and non-fundamental components. There are three popular methods for calculating apparent power, and at time of writing (2014) discussions are going on to close the gap between them:

**Classical**

The Classical method knows two flavours: arithmetic and vectorial.

The **arithmetic method** uses the mathematical sum of the apparent phase powers. This is very simple, but is incorrect with harmonics or unbalance.

The **vectorial method** uses the vector sum of the apparent phase powers. This leads generally to better results but still does not fully account for the harmonics and unbalance apparent power components.

**IEEE-1459**

IEEE 1459 replaces the unbalanced system with a virtual balanced system, called the effective system, which has the same overall external losses to the network as the original unbalanced system. This needs considerable calculations and some assumptions on the amount of unbalance present. Once we know the effective replacement system we can perform the apparent power calculations and analysis on this balanced replacement. This is simpler, but we lose the details of the original system. (The 430 Series II uses the effective replacement system calculations when switched to IEEE mode).
Unified

The UPM method instead uses the actual unbalanced system to calculate the apparent power. Its calculation of apparent power is based on the actual instantaneous power components in the system by applying a simple mathematical transformation. This leads to a calculation formula similar to the Buchholz method.

To summarize, all methods produce the same result when there is very little harmonic and unbalance disturbance. With significant harmonic or unbalance disturbance, the result from the methods will differ.

The Classical arithmetic approach just ignores harmonics and unbalance and so gives wrong results. The Classical vector sum gives better results for unbalanced sine-wave systems, but still misses important non-fundamental apparent power components.

The IEEE and Unified methods calculate apparent power for systems with significant unbalance and harmonic effects. The difference between IEEE 1459 and Unified results lies in what is included in apparent power and what is not. In general, we shall see that IEEE 1459 produces higher numbers for apparent power than the Unified method in the presence of large neutral currents. Under conditions of significant unbalance and high distortion, this can lead to over-specifying the system and hence wasting money.
Figure 3.4.4 three-phase power – unbalance – Classical calculation methods

3.4.4 unbalanced sine-wave – apparent power the Classical way ($S_a$ and $S_v$)

The classical calculation of apparent power knows two variations: the arithmetic sum and the vector sum.

**Arithmetic sum**

This method simply sums the individual phase apparent powers. For a three wire configuration it calculates the apparent power for each of the two wattmeter sections and adds them together. A disadvantage of this method is that the relation $S_a^2 = P^2 + Q^2$ is no longer valid on the system level.

As we have seen before, this method of simply adding only works for pure sine-wave systems with no unbalance.

**Vector sum**

This method finds the system apparent power by using the system active and reactive powers, which can be found just by adding the individual phase contributions. Here, the relation $S_v^2 = P^2 + Q^2$ is at the heart of the calculation. As we have seen in the apparent power vector diagram of 3.4.2, this is a better method.

The $P$'s and $Q$'s can each be added and the resulting $S_v$ takes unbalance between the individual phases into account.

However, the apparent system power associated with unbalance is not included. As the reactive power $Q$ is only defined for the fundamental frequency, the vector sum method is unsuitable for systems with harmonic components.

So, both methods work well for balanced sine-wave systems. The vector sum works better for unbalanced sine-wave systems but still ignores some important apparent power components. And both methods ignore apparent power due to harmonic components.
IEEE-1459 apparent power

\[ S_e = 3 \cdot U_e \cdot I_e \]  \hspace{1cm} (full effective apparent power)

\[ S_{e1} = 3 \cdot U_{e1} \cdot I_{e1} \]  \hspace{1cm} (fundamental effective apparent power)

**Four-wire WYE:**

\[ U_e = \sqrt{\frac{(3.(U_A^2 + U_B^2 + U_C^2) + U_{AB}^2 + U_{BC}^2 + U_{CA}^2)/18)}{}} \]

\[ U_{e1} = \sqrt{\frac{(3.(I_{1A}^2 + I_{1B}^2 + I_{1C}^2))/3)}{}} \]

**Three-wire DELTA:**

\[ U_e = \sqrt{\frac{(U_{AB}^2 + U_{BC}^2 + U_{CA}^2)}/9)} \]

\[ U_{e1} = \sqrt{\frac{(I_{1A}^2 + I_{1B}^2 + I_{1C}^2))/3)}{}} \]

Remark 1): As we have no knowledge of the (virtual) phase voltages inside the Δ-configuration, we have to use an approximation

---

3.4.5 unbalanced sine-wave – apparent power the IEEE 1459 way (\( S_e \) and \( S_{e1} \))

IEEE 1459 first calculates a symmetrical balanced replacement system (not shown). Then, it says – quite simply – that the effective apparent power \( S_e \) is \( 3 \times \) effective voltage \( \times \) effective current.

And the same for the effective fundamental apparent power \( S_{e1} \).

**Four-wire wye**

The calculations for the four-wire system involve line voltages, line-line voltages and line currents including the neutral current. Note that the neutral current is multiplied by all the line and line-line voltages. It therefore plays a significant role here. However, it seems illogical for apparent power to associate the line and line-line voltages with the neutral current, since this does not happen in the original, physical circuit. It seems to overstate the importance of the neutral current, and is a major reason for the differences between IEEE 1459 and the Unified approach.

**Three-wire delta**

The calculations for the three-wire system involve line-line voltages and line currents. As we do not know the internal load values of the Δ-system when we look from the outside, we do not have enough information to make exact calculations. So, we have to assume the size of unbalance in this internal load to calculate the apparent power for a three-wire system. If we assume a load unbalance of ≤ 10%, we can calculate the apparent power from line-line voltage and line current values with an accuracy of ≈ 0.2%. This approximation is a general limitation for measuring apparent power in a Δ-system without knowing the load distribution.

With this assumption, the calculations are actually slightly simpler than the four-wire calculations because of course there is no neutral voltage and no neutral current. Here, the IEEE 1459 and Unified results largely agree.
Unified apparent power (UPM)

**four-wire WYE:**

\[
S = \sqrt{(U_A^2 + U_B^2 + U_C^2)(I_A^2 + I_B^2 + I_C^2)}
\]

\[
S_1 = \sqrt{(U_{1A}^2 + U_{1B}^2 + U_{1C}^2)(I_{1A}^2 + I_{1B}^2 + I_{1C}^2)}
\]

**three-wire DELTA:**

\[
S = \sqrt{(U_{AB}^2 + U_{BC}^2 + U_{CA}^2)(I_A^2 + I_B^2 + I_C^2)/3}
\]

\[
S_1 = \sqrt{(U_{1AB}^2 + U_{1BC}^2 + U_{1CA}^2)(I_{1A}^2 + I_{1B}^2 + I_{1C}^2)/3}
\]

Remark): As we have no knowledge of the phase-phase currents inside the \( \Delta \)-configuration we have to use an approximation.

**Figure 3.4.6** three-phase power – unbalance – UPM calculation method (Buchholz)

### 3.4.6 unbalanced sine-wave – apparent power the Unified way (\( S \) and \( S_1 \))

The Unified approach does not calculate an effective replacement system, but instead calculates the apparent power from (a transformation of) the actual instantaneous power components.

**Four-wire wye**

The Unified approach finds \( S \) for an unbalanced system using Buchholz method using a sum of the phase voltage squares times the sum of the phase current squares. The effect of the neutral current is already included in the phase currents. Compared to the approach of the power vector diagram of 3.4.2, this gives more accurate results for apparent power when harmonics and unbalance are present.

Notice that according UPM the neutral current effect is already present in the phase currents. So the neutral current is not separately multiplied with the phase voltages (like in IEEE-1459), which can give different results to the IEEE-1459 approach.

There is a corresponding equation for the fundamental apparent power \( S_1 \).

**Three-wire delta**

For a three-wire system, we construct a similar equation but using the line-line voltages and the line currents, again according to the Buchholz formulae for apparent power. As we cannot measure the line-line currents from the outside we again have to make certain assumptions. Assuming a load unbalance \( \leq 10\% \), we can once more calculate the apparent power from line-line voltages and line currents with an accuracy of \( \approx 0.2\% \).
unbalanced system decomposition

A three-phase unbalanced phasor system can be decomposed into three balanced phasor systems:

- three-phase Positive Sequence System
- three-phase Negative Sequence System
- single-phase Zero Sequence System

(C.L. Fortescue 1918)

decomposing the fundamental components:

\[ U_{1A}, U_{1B}, U_{1C} \Rightarrow U^+, U^-, U^0 \]
\[ I_{1A}, I_{1B}, I_{1C} \Rightarrow I^+, I^-, I^0 \]

Figure 3.5.1  three-phase power – unbalance – decomposition in symmetrical components

3.5  symmetrical components

3.5.1  symmetrical components – introduction

To continue the unbalance analysis in more detail, we now decompose the unbalanced three-phase system into three balanced (symmetrical) phasor systems: a three-phase positive sequence, a three-phase negative sequence and a single-phase zero sequence. A phasor system is, again, a vector diagram with vectors representing the magnitude and phase angle of each individual phase voltage or current.

You can decompose the fundamental voltages to derive the positive-, negative- and zero-sequence voltages. And similar for the fundamental currents.

We have said that the Unified method decomposes the unbalanced system into its symmetrical components. So what are the symmetrical components?
3.5.2 symmetrical components – vector view

These three components (positive sequence, negative sequence and zero sequence) are all balanced: the same for each phase. The claim (originally made by C.L. Fortescue in 1918) is that we can construct the unbalanced system on the right from the three balanced systems on the left.

We start off from the fundamentals, with the same length and the same angles. The positive sequence is ABC, the negative sequence is ACB. (By convention, vector diagrams always turn anti-clockwise, corresponding with the ever increasing phase angle over time. Note, though, that the vectors of the positive sequence again cross the X-axis in order A, B, C while those of the negative sequence cross the X-axis in order C, B, A). For a balanced system, the negative- and zero-sequence values are zero and the vector sum is symmetrical.

The unbalanced system vectors

For an unbalanced system you start with the blue vector, then add the green and then the red vectors. They nicely form the vector $A_1 = A^0 + A^- + A^+$ (and the same for $B_1$ and $C_1$), showing how you can form an unbalanced system from three symmetrical components. The resulting unbalanced system vector, like the sequence vectors, rotates at 60 Hz around the origin (shown as a black dot). Unlike the symmetrical sequence vectors, though, the phases have different lengths and different phase angles.

Again this is illustrative rather than a strict mathematical proof. It shows, though, that you can separate out the three complicated rotating vectors $A_1$, $B_1$ and $C_1$ of your actual system into symmetrical positive-, negative- and zero-sequence vectors.
symmetrical powers

voltage: \( U_1^+, U_1^-, U_1^0 \)
current: \( I_1^+, I_1^-, I_1^0, \varphi_1^+, \varphi_1^-, \varphi_1^0 \)

Using positive sequence components:

active power: \( P_1^+ = 3 \cdot U_1^+ \cdot I_1^+ \cos(\varphi_1^+) \) (W)
reactive power: \( Q_1^+ = 3 \cdot U_1^+ \cdot I_1^+ \sin(\varphi_1^+) \) (var)
apparent power: \( S_1^+ = 3 \cdot U_1^+ \cdot I_1^+ \) (VA)

3.5.3 symmetrical components – introducing symmetrical powers

So we can decompose the fundamental voltages in your system into positive-, negative- and zero-sequence voltages. Similar for the currents and the phase angles (the angles between the symmetrical voltages and currents). Because these are symmetrical components, we can now use the old Classical method. Because we decompose the fundamental voltages and currents only, we deal with the harmonics later.

In this tutorial we use symbols like \( U_1^+ \) and \( I_1^+ \). The subscript \( 1 \) reminds us these are fundamental frequency values, the superscript \( ^+ \), \( ^- \) or \( ^0 \) stands for positive sequence, negative sequence or zero sequence.

Using positive-sequence components

The positive-sequence active power is three times the positive-sequence voltage and current multiplied by the relevant \( \cos \varphi \). This is the power that makes a motor deliver torque.

Similarly for the positive-sequence reactive power, but with \( \sin \varphi \). This power has no useful effect, but must still be carried by the system.

The positive-sequence apparent power is simply three times the positive-sequence voltage times the current. We can carry out a similar process for the negative sequence and the zero sequence to give nine equations in all.

So, using this transformation we can split the total powers into the component that contributes to the efficient transfer of energy (the positive sequence component) and the two components that cause losses (the negative and zero sequence components). The positive sequence component gives torque to the axle, while the negative sequence component acts as a brake. The zero sequence component moves the whole system up or down with respect to earth or reference, and mainly generates heat.
3.5.4 symmetrical components – current waveforms

The example on the next few pages helps to explain the symmetrical components method. The system has balanced voltages and unbalanced currents, both in amplitude and phase angle. Unbalanced currents (caused by an unbalanced load) are again more common than unbalanced voltages (caused by an unbalanced supply).

The top graph shows the balanced voltages. As unbalance is a property of the fundamental components, we will consider the fundamental voltages and currents only. Unbalance is one cause of neutral current. The other is harmonic currents, and this example assumes that there are no harmonic components.

The middle graph shows the unbalanced currents. Notice that, besides the amplitude differences, the phase angles are not nicely 120° spaced. So, we have both amplitude unbalance and phase unbalance.

This set of unbalanced currents does not add up to zero, so it produces a neutral current shown in the lower graph. Notice that the fundamental neutral current has an amplitude (RMS value) and a phase angle.

Figure 3.5.4 three-phase power – symmetrical components – example with unbalanced currents

<table>
<thead>
<tr>
<th>balanced voltages:</th>
</tr>
</thead>
<tbody>
<tr>
<td>$U_{1A} = U_{1B} = U_{1C} = 120V$</td>
</tr>
<tr>
<td>$\angle 0^\circ$, $\angle -120^\circ$, $\angle -240^\circ$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>unbalanced currents:</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_{1A} = 5 , A \angle -30^\circ$</td>
</tr>
<tr>
<td>$I_{1B} = 6 , A \angle -160^\circ$</td>
</tr>
<tr>
<td>$I_{1C} = 4 , A \angle -260^\circ$</td>
</tr>
</tbody>
</table>

neutral current:

\[
i_{1n}(t) = i_{1A}(t) + i_{1B}(t) + i_{1C}(t)
\]

\[I_{1n} = 2.09 \, A \angle -163^\circ\]
### 3.5.5 Symmetrical Components – The Symmetrical Transformation

With the fundamental voltages and currents as inputs, we can perform the symmetrical transformation. For this tutorial we consider the actual transformation as a black box. We input the amplitude and angle of the fundamental phase voltages and currents, and we get the amplitude and angle of the symmetrical voltages and currents as an output.

The benefit of finding the symmetrical voltages (currents) is that they tell you the real and imaginary parts of the three-phase signals – and so the effects on your power system. Adding the positive, negative and zero-sequence voltages (currents) on each phase gets you back to the original unbalanced voltages (currents) on each phase plus neutral.

Notice that the voltage has a positive-sequence component only, because the fundamental phase voltages are balanced. The current has positive, negative and zero-sequence components.

The **negative-sequence and zero-sequence ratios** indicate the amount of unbalance. These are the values of negative and zero components divided by the positive component expressed as a percentage. In our example these ratios are 0% for the voltage (because these are balanced), 16.6% for the negative-sequence current ratio and 14.1% for the zero-sequence current ratio (these ratios are defined as a measure for unbalance in standard IEC 61000-4-30: Power Quality Measurement Methods; maximum allowable values are given in the EN50160 Power Quality Standard).

#### Symmetrical Transformation Example

<table>
<thead>
<tr>
<th>Fundamental Phase Voltages:</th>
<th>Symmetrical Voltages:</th>
</tr>
</thead>
<tbody>
<tr>
<td>$U_{1A}$: 120V∠0°</td>
<td>$U_1^+$: 120V∠0°</td>
</tr>
<tr>
<td>$U_{1B}$: 120V∠-120°</td>
<td>$U_1^-$: 0V∠0°</td>
</tr>
<tr>
<td>$U_{1C}$: 120V∠-240°</td>
<td>$U_1^0$: 0V∠0°</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Fundamental Phase Currents:</th>
<th>Symmetrical Currents:</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_{1A}$: 5.0A∠-30°</td>
<td>$I_1^+$: 4.95A∠-31°</td>
</tr>
<tr>
<td>$I_{1B}$: 4.0A∠160°</td>
<td>$I_1^-$: 0.82A∠+20°</td>
</tr>
<tr>
<td>$I_{1C}$: 6.0A∠260°</td>
<td>$I_1^0$: 0.70A∠-163°</td>
</tr>
</tbody>
</table>

#### Analysis

<table>
<thead>
<tr>
<th>Negative Sequence Ratio</th>
<th>Calculation</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>%U$^-$</td>
<td>$(</td>
<td>U_1^-</td>
</tr>
<tr>
<td>%I$^-$</td>
<td>$(</td>
<td>I_1^-</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Zero Sequence Ratio</th>
<th>Calculation</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>%U$^0$</td>
<td>$(</td>
<td>U_1^0</td>
</tr>
<tr>
<td>%I$^0$</td>
<td>$(</td>
<td>I_1^0</td>
</tr>
</tbody>
</table>
symmetrical components

![Symmetrical Components Diagram](image)

### 3.5.6 symmetrical components – the transformed current waveforms

Once we have found the amplitude and phase of the symmetrical sequence components using the symmetrical transformation, we can draw their waveforms.

**Positive sequence**

For the positive-sequence current waveform, the values we get from the transformation are for phase A, as this is the phase we use as a reference for our horizontal scale (we have defined $\varphi_{U1A} = 0^\circ$). As the positive-sequence currents are from a balanced system, we find phase B and phase C by simply shifting the phase-A waveform by $-120^\circ$ and $-240^\circ$. The positive-sequence current transports the useful power to the load. It has the same A, B, C sequence as the original currents, but it contains no unbalance components.

**Negative sequence**

We find the negative-sequence current waveform in a similar way, using the negative-sequence values from the transformation (but note the difference in scale of the waveforms). The difference to the positive sequence is that the phase-B waveform is now $-240^\circ$ shifted and the phase-C is $-120^\circ$ shifted because this is the negative A-C-B sequence.

**Zero sequence**

And we similarly find the zero-sequence current waveform using the zero-sequence values from the transformation. The difference here is that the three waveforms for phase A, B and C are not shifted in respect to each other. The zero-sequence waveforms have all the same phase: they lie on top of each other.
The negative- and zero-sequence currents are the result of the unbalance in the original currents. They do not transport useful power to most loads, and cause braking torques and dissipate heat in rotating machines.

**Applying the symmetrical transformation**

By using the symmetrical transformation, we can replace the unbalanced phase currents by the sum of the three symmetrical phase currents (which makes for simpler calculations):

\[
I_{1A}(t) = I_{1A}^+(t) + I_{1A}^-(t) + I_{1A}^0(t)
\]
\[
I_{1B}(t) = I_{1B}^+(t) + I_{1B}^-(t) + I_{1B}^0(t)
\]
\[
I_{1C}(t) = I_{1C}^+(t) + I_{1C}^-(t) + I_{1C}^0(t)
\]

We saw before that the neutral current is the sum of the three phase currents:

\[
I_{1n}(t) = I_{1A}(t) + I_{1B}(t) + I_{1C}(t)
\]

Now substitute the sum of the symmetrical currents for the phase currents:

\[
I_{1n}(t) = I_{1A}^+(t) + I_{1A}^-(t) + I_{1A}^0(t) + I_{1B}^+(t) + I_{1B}^-(t) + I_{1B}^0(t) + I_{1C}^+(t) + I_{1C}^-(t) + I_{1C}^0(t)
\]

But the positive-sequence currents have the same amplitude and are exactly 120° shifted, so their sum is zero. The same is true for the negative-sequence currents. They are both balanced three-phase currents:

\[
I_{1A}^+(t) + I_{1B}^+(t) + I_{1C}^+(t) = 0
\]
\[
I_{1A}^-(t) + I_{1B}^-(t) + I_{1C}^-(t) = 0
\]

The only term that remains is the sum of the three zero currents. These don’t cancel because they all have the same phase:

\[
I_{1n}(t) = I_{1A}^0(t) + I_{1B}^0(t) + I_{1C}^0(t)
\]

And since:

\[
I_{1A}^0(t) = I_{1B}^0(t) = I_{1C}^0(t)
\]

\[
I_{1n}(t) = 3 . I_{1}^0(t)
\]

And the fundamental neutral current is simply three times the zero-sequence current.

So, we can now draw the sequence waveforms, and do our calculations on symmetrical, balanced waveforms. And the accumulated results have the same values as for the unbalanced system you’re studying.

Now to check that statement ...
symmetrical components

3.5.7 symmetrical components – checking the currents

If what we said in the previous slide is true, then adding the symmetrical currents per phase must result in the original unbalanced phase currents and three times the zero-sequence current must give the neutral current.

To check if this is really true, we let Excel add and plot the results. The sum of the symmetrical currents is shown in the top graph, and three times the zero-sequence current is shown in the bottom graph. Compare these with the graphs of the original unbalanced currents in figure 3.5.4.

This demonstrates graphically C.L. Fortescue’s theorem: A set of three unbalanced voltages or currents can be represented by three sets of balanced voltages or currents: a positive set (phase sequence A, B, C), a negative set (phase sequence A, C, B) and a zero set (all the same phase).
3.5.8 symmetrical components – active and reactive positive-sequence currents

As before, the positive-sequence current is the most important one because it transports the useful power to the load. But not all the positive-sequence current actually converts electrical energy into another form.

Active and reactive positive-sequence currents

If the load is not purely resistive, part of the positive-sequence current is active current and part is reactive current. If we want to know the useful current (current that is effectively used for the energy conversion process), we must split the positive-sequence current into an active and reactive part. This is done the same way as in any balanced three phase system:

\[ I_{1a}^+ = I_1^+ \cos(\omega t_1+ - \omega t_1+) \]

\[ I_{1r}^+ = I_1^+ \sin(\omega t_1+ - \omega t_1+) \]

The active positive-sequence current is in phase with the positive-sequence voltage and results in the active positive-sequence power \( P_1^+ \) (W). This is the electrical power that is converted into mechanical power in a motor

\[ P_1^+ = I_{1a}^+ V_1^+ \]

The reactive positive-sequence current is 90° out of phase with the positive-sequence voltage and results in the reactive positive-sequence power \( Q_1^+ \) (var). This is the electrical power that bounces between the source and the load without transporting any useful energy, but causing network load and transport losses.

\[ Q_1^+ = I_{1r}^+ V_1^+ \]

The reactive positive-sequence power \( Q_1^+ \) (which equals \( Q_1 \) in a balanced system) is in general the only reactive power that is discussed. Full reactive power and non-fundamental reactive power have a mathematical definition, but they are more difficult to understand and interpret.
3.5.9 symmetrical components – a summary of the powers

This slide compares the fundamental powers against the (fundamental) positive-sequence and unbalance powers. This page talks only about fundamental powers – we are ignoring harmonics.

Look first at the fundamental and the positive-sequence powers. Notice is that there is no difference between the active powers, and no difference between the reactive powers. This is because we have a balanced voltage system so the fundamental and positive-sequence voltages are the same (the negative- and zero-sequence voltages are zero). There is no unbalance voltage, hence no active unbalance power even though there is current unbalance.

The difference comes in the apparent power. The fundamental apparent power includes the unbalance apparent power, and so is higher than the positive-sequence apparent power which does not.

Now look at the unbalance powers. The active unbalance power is the fundamental active minus the positive-sequence active: the larger the difference, the greater the unbalance. Here, as we saw above, the unbalance active power is zero.

We find the unbalance apparent power by taking the difference of the squared values of the fundamental and positive-sequence apparent powers. This power increases the load on the network.

The amount of system unbalance can be expressed by the Unbalance Factor $UF = S_U/S_1^+$. 

We’ll now look at some examples, comparing the results from Classical, IEEE 1459 and Unified approaches.
Some simple examples now to demonstrate that unbalance, like harmonics, disturbs the system. People know that transformers should be derated for harmonics, but few realize this is true for unbalance too. These examples show that it is. We start with resistive loads.

This first example shows three 1200 W lamps. The system is symmetrical, with all 10 A currents (which are positive-sequence currents) cancelling each other out at point n so there is no current into neutral. Classical, Unified and IEEE 1459 all agree that the active power in each phase (top) is 1200 W, reactive power is 0 W, and so the apparent power in each phase is also 1200 W.

Although we’ve only talked about the positive-, negative- and zero-sequence currents and powers in relation to unbalanced inductive loads, they of course also apply here, to unbalanced resistive loads. Here, the positive-sequence currents are each 10 A, and the negative- and zero-sequence currents are 0 A.
3.6.2 unbalance examples – effects of a burned-out lamp

Say now that one lamp burns out, so the active system power (bottom) is now only the power in the two remaining lamps, or 2400 W.

The apparent power from the Unified method is now not 2400 VA of the Classical method, but 2939 VA. The difference goes into a large value of unbalance apparent power (the Unified method calculates it as 1697 VA). That is because the three-phase generator is determined to deliver three-phase power with the same phase voltages and the same currents.

Because of that, when one lamp fails the 10 A current in that phase drops to zero and instead you suddenly have 10 A flowing in neutral, which is a bit of a surprise. The fact of a neutral current flowing is in itself already a sign that all is not well: neutral is a reference point and is not supposed to carry current except in fault conditions.

The various methods differ in the way they calculate the apparent and unbalance apparent powers $S$ and $S_{1U}$. The difference is mainly caused by the way the 10 A neutral current is included in $S$. And since $S_{1U}^2 = S^2 - (S_1^+)^2$ a different $S$ leads to a different unbalance apparent power $S_{1U}!$ (Both Unified and IEEE 1459 methods agree on $S_1^+$).

The Classical method just adds the phase apparent powers $S_x$ arithmetically, and ignores the influence of unbalance. That suggests that the unbalance has no influence on $S$ and $S_y$, which is clearly wrong under these conditions.

The Unified approach uses Buchholz’s treatment of apparent power (the maximum possible active power transmitted to a load for a given voltage waveform), and adds the apparent power in the neutral conductor. Now, the positive-sequence currents (not shown in the diagram) are each 6.67 A, the negative-sequence currents are each 3.33 A, and the zero-sequence currents are each a third of the fundamental neutral current, or 3.33 A. Because the neutral current is multiplied only by the voltage across the neutral
conductor to find the apparent power component, this only has a moderate influence on S. That gives the 1697 VA unbalance power $S_{uU}$ of the last diagram.

**IEEE 1459** introduces the effective power. It is mathematically sound and very complete, but hard to read for a non-mathematician and some of its parameters are difficult to link with physical phenomena. It multiplies the neutral current by all the phase and phase-phase voltages, which results in a heavy influence of the neutral current on $S_e$ and $S_{e1}$. For that reason it gives a high value of $S$ and hence a high value of 2683 VA for unbalance power in this example.

Which is the best approach? That depends on your point of view. For an engineer it is hard to see why a current (the neutral current, which does not flow through the branches) has to be multiplied by a voltage that has no relation to it, as in IEEE 1459. The unbalance power information obtained by the Unified method allows us to use passive unbalance compensation techniques, while the IEEE 1459 method leads to unbalance compensation that consumes more overall active power.

The 430 Series II actually allows a choice between Classical (arithmetic), Unified, and IEEE 1459 results.
3.6.3 unbalance examples – effects of poor installation

Let’s say that you had a lazy installer who installed two lamps in the A phase rather than one in A and one in C (the wires were easier to reach or too short and it was nearly weekend).

Two plus one lamps

Connect the lamps in parallel in the A phase rather than the C phase and you double the current to 20 A. The total active system power is the same 3600 W but now the apparent power calculated by Unified and IEEE 1459 is much higher.

The Classical calculations again ignore the unbalance power altogether. It has a large value of 2939 VA for Unified calculations and an even higher 4648 VA for IEEE 1459. Once again, this high figure seems to come from giving the neutral current a heavy weighting.

The positive-sequence currents are each now 10.0 A, the negative-sequence currents are each 5.77 A, and the zero-sequence currents are each 5.77 A.
3.6.4 unbalance examples – effects of extremely poor installation

The third example is even worse, with three lamps connected to the same phase. This time we have 30 A in one phase, and a very large unbalance power component.

Here again, the Classical calculations ignore unbalance apparent power.

The Unified method calculates an unbalance power of 5091 VA – much larger than the 3600 W active power. Once again this happens because the generator is determined to supply a balanced three-phase system. The positive-sequence currents are each now 10.0 A, the negative-sequence currents are each 10.0 A, and the zero-sequence currents are each 10.0 A.

The IEEE 1459 prediction is 8050 VA – much higher than even the Unified calculations for the same reason as before.

Note: For more details on IEEE versus UPM apparent power see appendix, chapter 5.1.
3.7 combined three-phase power (unbalance and harmonics)

3.7.1 combined power – unbalance and harmonic distortion

Figure 3.5.9 summarized the fundamental, positive sequence and unbalance powers. What happens if we add harmonics?

The full P, Q and S powers include all signal components, including unbalance and harmonics. Taking out all the harmonics leaves the fundamental components, and taking out the unbalance leaves the positive-sequence components. We can write the relationships fairly simply to find the actual harmonic and unbalance powers.

**Harmonic powers**

The harmonic active power is the full active power minus the fundamental active power. The harmonic reactive power is again of no interest, it is contained in the non-fundamental apparent power $S_N$. This non-fundamental apparent power is again the vector subtraction of the full apparent power minus the fundamental apparent power.

**Unbalance powers**

The unbalance active power is again similarly the fundamental active power minus the positive-sequence active power. (Unbalance, and positive-, negative- and zero-sequence powers are always fundamental). The unbalance reactive power is, like the harmonic reactive power, of no interest. And the unbalanced apparent power is the vector subtraction of the fundamental apparent power minus the positive-sequence apparent power.

So, once we have found the full, fundamental and positive-sequence components of P, Q and S we can quantify harmonic and unbalance effects. Then, we can correct them.
You can think of the different types of power as similar to the Matryoshka (Babushka) Russian Dolls which fit into each other.

Largest is the full power (apparent showing what the utility supplies, and active showing what you actually use). This full power separates out into the fundamental powers and (on the far right) the wasted harmonic powers.

The fundamental powers separate out at system level into positive-sequence and unbalance powers (negative and zero sequence). Of all these, you generally pay for the full apparent power $S$, but only actually use the fundamental active positive-sequence power $P_1^+$. 
3.7.3 combined power – example with unbalance and harmonic distortion

Now look at unbalance in a general three-phase system with a three-phase motor driven by an electronic circuit, so now including an inductive load with harmonics. We’ve symbolized the unbalance by showing different sizes for the loads. The total is three times 600 VA with a cosφ of 0.866 again.

Phase currents

This time we’ll assume the A, B and C phase currents are 5, 6 and 4 A respectively, and we’ll assume the third-harmonic currents to be 1.4, 1.7 and 1.1 A respectively. Phase angles for the fundamental current of -30°, -160° and -260° and for the third-harmonic current of 0°, -120° and -240°.

This is also as complex a system as you will meet – the only addition would be to add voltage unbalance and voltage harmonics (which 430 Series II can incidentally handle).
3.7.4 combined power – unbalance vector diagram

Note that the fundamental voltages for this last example are still nice and balanced. That is one great feature of the vector diagram – it shows at a glance which components are balanced and which not, the sizes of phase voltages and their respective currents, and the angles between them.

The currents are more complicated, with different magnitudes and different angles from the usual 0°, -120° and -240°, meaning a large unbalance current into the neutral.

We could draw a similar diagram for the third harmonic voltages and currents, but in general this analysis is only done for the fundamental components because they are the ones transporting useful energy.
3.7.5 combined power – example with unbalance and harmonics

The calculations for these unbalanced inductive loads plus harmonics are as we explained before, and we’ve here used Excel to calculate the results. The individual figures themselves are not important, just concentrate on the differences between Classical, IEEE and Unified approaches.

Active power

The active power for each phase is again the average of the voltage over time times the current over time – largest for phase B (carrying 6 A), smaller for phase A (5 A) and smallest for phase C (4 A). All three approaches agree that the active power at system level is the simple addition of the phases, or 1522 W.

Fundamental active power

The fundamental active powers are again the RMS values of fundamental voltages and currents times the cosine of the angle between them. That gives the same total as the active power in all three approaches because there is no harmonic power – there is harmonic current but no harmonic voltage and so no harmonic power.

Fundamental reactive power

The calculation is as for the fundamental active power, but instead uses the sine of the angle.

Apparent power

With the apparent power, we see the differences between the Classical, Unified and IEEE 1459 methods. The calculations for the individual phases are the same, but the system-level results are not. And it is the system-level apparent power that is the key figure since that is what the utility sees – and usually charges for.

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Apparent power

With the apparent power, we see the differences between the Classical, Unified and IEEE 1459 methods. The calculations for the individual phases are the same, but the system-level results are not. And it is the system-level apparent power that is the key figure since that is what the utility sees – and usually charges for.
The **Classical** result uses simple addition of the phases. The **Unified** method uses the sum of the squares of the voltages times the sum of squares of the currents. **IEEE 1459** uses the effective apparent power.

Note the difference in results, particularly the low result of the Classical method. The Unified method gives a larger figure, with IEEE 1459 calculating the highest values for unbalance and harmonic influences. The differences again have a lot to do with the way the neutral current is treated in the calculations. Globally this means:

- Classical: ignores the neutral current
- Unified: neutral current x voltage across the neutral conductor
- IEEE: neutral current x all phase and phase-phase voltages

The major differences are highlighted here and on the next diagram by outlining the boxes in red. The 430 Series II helps show these differences by being able to switch between Classical, IEEE 1459 and Unified calculations.
The last three entries of the previous table gave the full apparent powers – so including fundamental and harmonics. We now separate out the effects of harmonics and unbalance. We therefore break the full apparent powers down into the fundamental, non-fundamental and unbalance apparent powers. The fundamental apparent powers for the individual phases are the fundamental voltage multiplied by the fundamental current for each phase, being 120 V multiplied by 5 A, 6 A and 4 A.

First, find the system-level fundamental apparent power (first three lines). The Classical calculations just add the three phases but they underestimate the true system values, and Unified and IEEE 1459 again give different results.

Now, find the harmonics. The non-fundamental apparent power for each phase (next three lines) starts from the Classical calculations just adding the three phases. Again, Unified and IEEE 1459 calculations give higher results. As this example generates a large neutral current $I_n = 4.7$A (2.1A fundamental and 4.2A 3rd harmonic), the IEEE apparent powers $S_{eN}$ and $S_{1eU}$ are heavily influenced, leading to large numbers for $S_{eN}$ and $S_{1eU}$.

And finally, find the unbalance. The apparent fundamental unbalance power (last two lines) does not appear in the Classical approach, but does in Unified and IEEE 1459. The values differ because of the differences in fundamental apparent power. Note that there is no unbalance power in the individual phases!

---

### Table: Three-Phase Power - Combined Power - Example Power Calculations (Contd.)

<table>
<thead>
<tr>
<th>Formula</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>Sys</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_{1X} = U_{1X} \cdot I_{1X}$</td>
<td>Class</td>
<td>600.0</td>
<td>720.0</td>
<td>480.0</td>
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<tr>
<td>$S_1 = \sqrt{\left(\Sigma U_{1X}^2 \cdot \Sigma I_{1X}^2\right)}$</td>
<td>Uni</td>
<td>,</td>
<td>,</td>
<td>,</td>
</tr>
<tr>
<td>$S_{e1} = 3 \cdot U_{e1} \cdot I_{e1}$</td>
<td>IEEE</td>
<td>,</td>
<td>,</td>
<td>,</td>
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<td>$S_{NX} = \sqrt{\left(S_X^2 - S_{1X}^2\right)}$</td>
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<td>168.0</td>
<td>204.0</td>
<td>132.0</td>
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<td>,</td>
<td>,</td>
<td>,</td>
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<td>$S_{eN} = \sqrt{\left(S_{e}^2 - S_{e1}^2\right)}$</td>
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<td>,</td>
<td>,</td>
<td>,</td>
</tr>
<tr>
<td>$S_{1U} = \sqrt{\left(S_{1}^2 - (S_1^*)^2\right)}$</td>
<td>Uni</td>
<td>n.a.</td>
<td>n.a.</td>
<td>n.a.</td>
</tr>
<tr>
<td>$S_{e1U} = \sqrt{\left(S_{e1}^2 - (S_{1}^*)^2\right)}$</td>
<td>IEEE</td>
<td>n.a.</td>
<td>n.a.</td>
<td>n.a.</td>
</tr>
</tbody>
</table>
3.8 neutral currents

3.8.1 neutral currents – due to unbalance

Ideally (balanced with no harmonics) in a Y configuration the three phase currents add up to zero at the star point at any point in time. This means that even though there are reactive phase currents, these also cancel out and there is no neutral current, so \( i_A(t) + i_B(t) + i_C(t) = i_n(t) = 0 \). This is the result of three pure sine waves of equal size, each shifted 120° to each other (top waveform).

There are situations however where this is not true:

- if the amplitudes of the fundamental phase currents \( I_{1A} \), \( I_{1B} \), \( I_{1C} \) are not equal (amplitude unbalance),
- if the phase angles of the fundamental phase currents \( \phi_{1A} \), \( \phi_{1B} \), \( \phi_{1C} \) are not shifted by 120° to each other (phase unbalance),
- if the phase currents \( i_A(t) \), \( i_B(t) \), \( i_C(t) \) are not pure sine waves but contain harmonics (harmonic distortion).

Here, a neutral current may flow. We will look at the two unbalance situations first (amplitude and phase unbalance but no harmonic component, middle waveform). If we draw \( i_A(t) \), \( i_B(t) \), \( i_C(t) \) in a graph and sum them point by point (in an Excel sheet) the resulting waveform \( i_n(t) \) is not zero but a sine wave of the fundamental frequency (bottom waveform). So:

- current unbalance produces a fundamental frequency neutral current, the size of which depends on the sizes of the amplitude and phase angle unbalance.

Note that voltage unbalance almost always leads to current unbalance (because the voltage generates the current) and so to neutral current.
3.8.2 neutral currents – due to harmonics

Now consider when the phase currents \( i_A(t) \), \( i_B(t) \), \( i_C(t) \) contain harmonic components. These components may add up or cancel out in the starpoint of the Y configuration, depending on the frequency and phase of the harmonics. A special case is the multiple of the third harmonics (\( I_{3A}, I_{3B}, I_{3C} \), ..., called triplen harmonics. These harmonics tend to add up in the starpoint of the Y configuration, causing large neutral currents.

Consider the example above with equal phase currents, neatly spaced at 120°. They are perfectly balanced, and each contains the same third harmonic current (middle waveform). This is still a balanced system, because for unbalance we only consider the fundamental component. If we add these perfectly similar, 120° spaced phase currents \( i_A(t) \), \( i_B(t) \), \( i_C(t) \) we find a large third harmonic neutral current \( i_{n}(t) \). Why do the phase currents not cancel each other out?

The answer is in the 120° shift: what is 120° for the fundamental is 360° for the third harmonic (2 x 360° for the sixth, 3 x 360° for the ninth, etc.) Looking at the bottom diagram of \( i_{n}(t) \), one period of the third harmonic fits exactly into 120° of the fundamental component. So instead of canceling each other out, the third harmonic components add up in the starpoint.

This is true for all multiple-of-three harmonics, and in general we can state:

- Multiple-of-three (triplen) harmonic currents tend to accumulate in the zero current.

Looking back, the example with the bridge rectifier circuit caused large third harmonic currents. Three-phase rectifiers used for large equipment can have the same problem of generating third harmonics. With the huge number of these rectifiers in the industrial environment, you can see the problem!

The way to avoid harmonic neutral currents is to avoid harmonic phase currents by using filters immediately in front of the equipment that causes them.
3.8.3 neutral currents – due to harmonics and unbalance

Combining unbalance (amplitude and phase angle) and third harmonic current components also causes neutral current. Here, too, this heats up the neutral conductor and the utility transformer at the feeding point. The conductors and transformers can therefore have to be far larger than without unbalance and harmonics. This is a significant waste of money.

The third harmonic component in the neutral current above is slightly less than in the previous example because the third-harmonic phase currents are not shifted by exactly 120° to each other. This has a slight cancelling effect!

The effects of large neutral currents can be disastrous (for example causing fires), and the neutral conductor needs to be specified for the maximum possible unbalanced current.

Note also the following practical issues:

- All the neutral and ground (or "earth") wires in a building are tied or linked together at the incoming service main breaker panel. This is the only place they should ever be tied together because it is "upstream" of all the fuses and/or circuit breakers protecting the hot (or "live") wires for the various circuits installed in the building.

- **Warning:** never assume that a neutral is safe to touch: it has to be checked with a voltmeter or a voltage indicator to be sure it is not "live".

- In buildings with a large number of installed personal computers, the neutral conductor can carry much higher currents than it was designed for, creating a potential fire hazard.
• For transformers feeding harmonic-producing loads, the eddy current loss in the windings is the dominant loss component. This eddy current loss increases proportional to the square of the harmonic current and the square of its corresponding harmonic number (\(I_H^2\cdot H^2\)). The total transformer loss in a fully loaded transformer supplying a nonlinear load is twice as high as for an equivalent linear load. This causes excessive transformer heating and degrades the insulation materials in the transformer, which eventually leads to transformer failure. (http://www.copperinfo.co.uk/power-quality/downloads/pub-144-harmonics-transformers-k-factors.pdf)

• All circuits containing capacitance and inductance have one or more resonant frequencies. When any of the resonant frequencies correspond to the harmonic frequency produced by nonlinear loads, harmonic resonance can occur. Voltage and current at the resonant frequency can be highly distorted, and this distortion can cause nuisance tripping in an electrical power system. That can ultimately result in production losses. (http://www.grainger.com/Grainger/static/supplylink-check-electrical-power-system-harmonics.html)

• For more details on sizing the neutral conductor see: (http://ecmweb.com/code-basics/characteristics-neutral-conductor).

• See also the National Electrical Code in the US: NEC Article 250 (see also the NEC Article 250 presentation given at http://fyi.uwex.edu/mrec/files/2011/04/W4.-Biesterveld-NEC-grounding-MREC2010.pdf).
three-phase power overview (Classical)

<table>
<thead>
<tr>
<th>POWER</th>
<th>full</th>
<th>fund.</th>
<th>harmonic</th>
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<tbody>
<tr>
<td>apparent</td>
<td>$S_a$</td>
<td>$S_{a1}$</td>
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<td>$P$</td>
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<td>$P_H$</td>
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<td>$Q_1$</td>
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<td>var</td>
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<tr>
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</table>

Figure 3.9.1  three-phase power – power summaries – Classical power parameters

3.9  power summaries

3.9.1  power summaries – Classical parameters

This summarizes the key powers and performance factors of the Classical approach – full, fundamental and harmonic powers. The subscript ‘a’ again stands for arithmetic.
3.9.2 power summaries – Unified parameters

Compare the Unified parameters, remembering that the apparent powers are the most important for the system load, together with their associated power factors.

Apparent, active and reactive powers

The Unified method replaces the Classical arithmetic apparent power \( S \) with the Buchholz apparent power \( S \) (no subscript). It adds two new flavours of fundamental value, both related to unbalance: fundamental positive-sequence power and fundamental unbalance power. Note that we only talk about the unbalance of fundamental components – this is the figure that needs compensating for – we don’t care about the unbalance of harmonic components, that is included in \( S_N \).

The Unified method adds positive-sequence active \( P_1^+ \) and positive-sequence reactive \( Q_1^+ \) powers.

Performance (line utilization) factors

The power factor and harmonic pollution parameters are similar to the Classical method, but with fundamental positive-sequence power factor instead of Displacement Power Factor. We have selected \( PF_1^+ \) here because this lines up with IEEE 1459. It excludes harmonics and unbalance. In the Unified theory, though, you can actually calculate:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Formula</th>
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<tbody>
<tr>
<td>Power Factor</td>
<td>( PF = P/S )</td>
</tr>
<tr>
<td>Displacement Power Factor</td>
<td>( DPF = P_1/S_1 )</td>
</tr>
<tr>
<td>Positive-Sequence Power Factor</td>
<td>( PF_1^+ = P_1^+/S_1^+ )</td>
</tr>
<tr>
<td>Efficiency Factor</td>
<td>( EF = P_1^+/S )</td>
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<tr>
<td>Harmonic Pollution Factor</td>
<td>( HP = S_N/S_1 )</td>
</tr>
<tr>
<td>Unbalance Factor</td>
<td>( UF = S_{1U}/S_1^+ )</td>
</tr>
</tbody>
</table>
One extra important measure is the Unbalance Factor, also shown in the table, which rates the unbalance power.

Just these three factors (Power Factor, Harmonic Pollution and Unbalance Factor) immediately show what is happening in a system.

**Note:** For more details on UPM power components see appendix, chapter 5.2.
3.9.3 power summaries – IEEE 1459 parameters

Now compare IEEE 1459 to Unified parameters, which mainly differ in the choice of symbols (although the relevant parameter values may of course also differ).

**Apparent, active and reactive powers**

IEEE 1459 uses *effective* apparent powers but otherwise is the same as for the Unified method.

Active and reactive powers are the same.

**Performance factors**

Here, too, the only difference in the power factors is the IEEE 1459’s use of effective power.
3.9.4 power summaries – power directions

To avoid confusion in “bi-directional” metering applications where power can flow from the source to the load and back again, IEEE 1459 includes a number of widely used naming conventions.

Active (W) and reactive power (var)

When active power (W) or reactive power (var) flows from the source through the metering point to the load we say it is delivered and has a positive (+) sign.

When active power (W) or reactive power (var) flows from the load through the metering point to the source we say it is received and has a negative (-) sign.

Apparent power (VA)

When active power (W) flows from the source through the metering point to:

- a resistive load (no var) the apparent power (VA) will act along the x axis with the active power (W),
- an inductive load (positive var) the current I lags the voltage U (voltage leads current in an inductor) and the apparent power (VA) will be in quadrant 1,
- a capacitive load (negative var) the current I leads the voltage U and the apparent power (VA) will be in quadrant 4,

When active power (W) flows from the load through the metering point to the source to:

- an inductive load (positive var) the current now leads the voltage and the apparent power (VA) will be in quadrant 2,
- a capacitive load (negative var) the current now lags the voltage and the apparent power (VA) will be in quadrant 3.
Note 1: As apparent power is the product of RMS values, it has no sign or defined direction, but depending on the direction of active and reactive power it is said to be in quadrant 1, 2, 3 or 4.

Note 2: Current is said to lead or lag voltage viewed from the perspective of the side supplying the active power, the source.

For more details see:  

Active or reactive power flowing from source to load is delivered and has a positive (+) sign.  
Active or reactive power flowing from load to source is received and has a negative (-) sign.  
Apparent power is always a positive number, it has no direction or sign but is said to be in a quadrant.
single-phase power losses

power that is generated and not put into useful work is LOSS.

\[
\begin{align*}
\text{generator} & \quad \text{load} \\
I & \quad r_{\text{line}}/2 \\
+ & \quad U_{\text{AC}} \\
- & \quad r_{\text{line}}/2
\end{align*}
\]

- line current: \( I^2 = I_a^2 + I_r^2 + I_H^2 \)
- line loss: \( P_j = I^2 \cdot r_{\text{line}} \)
- the line loss can be split into:
  - active loss: \( P_{ja} = I_a^2 \cdot r_{\text{line}} \)
  - reactive loss: \( P_{jr} = I_r^2 \cdot r_{\text{line}} \)
  - harmonic loss: \( P_{jH} = I_H^2 \cdot r_{\text{line}} \)
- harm. power: \( P_H = P - P_1 \)
- total loss: \( P_{\text{Loss}} = P_{ja} + P_{jr} + P_{jH} + P_H \)

**Figure 4.1.1** power loss – single-phase loss – line power loss

## 4 power loss

### 4.1 single-phase power loss

#### 4.1.1 single-phase losses – line power loss basics

You pay for all the electricity you use – whether wasted or not. Although reactive power (for example) in a circuit degrades your power factor, it doesn’t necessarily lead to losses – the power bounces between source and load. Unified theory however points out that the currents associated with the reactive power cause losses in line resistance (and transformer cores). These line losses can be appreciable because the total current in your system flows through the line resistance.

The total RMS AC line current into your system is the vector sum of active, reactive and harmonic currents. The total line loss \( P_j \) (joule, or heating, loss) is the \( I^2R \) loss. Much of this loss is easily prevented – once you know which equipment is causing which type of loss.

From the network side, you can only reduce the **active line loss** by increasing the thickness of copper wires or increasing the voltage (to reduce \( I^2R \) transmission losses), neither of which are normally economic. Distinguish this, though, from the total **active loss**, which comes from the equipment installed in your buildings. Good maintenance can ensure that electrical equipment works as efficiently as it can (this is where equipment like infra-red cameras come into their own, to find any heat spots indicating inefficiencies in motors and other equipment). Good purchasing reduces your future energy costs by choosing energy efficient equipment.

You can simply compensate for **reactive loss** with a capacitor. Then, rather than the power going to and from the utility supply into the inductor at each cycle, it goes to and from the capacitor.
You can reduce **harmonic line losses** by filtering the waveform before it passes across a long line. Then, similarly, the distortion appears across the filter rather than being transmitted to the utility. The filter can consist of just a capacitor or capacitor/inductor network, which will combine with the parasitic resistance and inductance/capacitance in the supply. Active power factor correction can reduce the harmonic currents further, always aiming to keep the power factor as close as possible to 1.

The **harmonic power loss** is actually the harmonic power $P_H = P - P_1$, although whether this is actually a loss or not again depends on the type of load. Most harmonic power is converted into heat, so for a heater it would not be a loss. For a motor or transformer it is certainly loss, and comes on top of the harmonic line loss.

The total line loss is the sum of all the individual line losses and the harmonic power.
4.2 three-phase power loss

4.2.1 three-phase loss – line power loss in practice

In the chain between a utility generating power and the end user consuming it, part of the generated power is lost. According to the IEC publication "Efficient Electrical Energy Transmission and Distribution" (IEC 2007), overall losses between power plant and user can easily be between 8% and 15%. Overhead lines account for about 2.5% of the loss, and the step-down substation for between 3% and 5%. On average, these losses are double those between the substation and the user.

The publication mentions three main causes:

1. the Joule effect, where energy is lost as heat in the conductor (a copper wire, for example)
2. magnetic losses, where energy dissipates into a magnetic field
3. the dielectric effect, where energy is absorbed in the insulating material.

This section will focus on cause 1: the line losses, and particularly the line losses at the user side of the system. These are losses which appear on users’ electricity bills and can be directly influenced by them. This does not mean that the other losses are less significant or important, but they are influenced by the utility rather than the user and are part of the electricity tariff.

The diagram shows the general line loss situation: currents flow through non-ideal conductors with a certain resistance. In general, the phase conductors will all have the same line resistance because they have equal length and cross sectional area. The neutral conductor however can have a different cross sectional area, and hence a different resistance. As the currents flow through the line resistances, heat is generated according to \( P_{\text{heat}} = I^2 R \). This is active power that is supplied by the source but lost to the load.
4.2.2 three-phase loss – line power loss from a user perspective

This diagram refines the general situation to focus on the user side of the problem. We have split the line section into two parts: a utility and a user part, with the metering point being the boundary. Losses to the left of the metering point will be part of the electricity tariff. They have to be paid for by users, but are outside their direct influence. Losses to the right of the metering point appear directly on a user’s electricity bill. They are part of the in-house installation and as such within the user’s responsibility.

The first question is: why should I worry about these losses? That depends on how perfect the in-house installation is. For a perfectly balanced load without phase shift or harmonic currents, the energy efficiency of the loads themselves will dominate the savings to be made. In practice, however, the situation can be far from perfect and a simple tool that indicates the power (and hence money) lost and where to start with improvements is often handy. On the Fluke 430 Series II, that is called the Power Loss Calculator.

The next part will summarize the three-phase line losses, and suggest practical applications with a real-world example.
### three-phase line loss — current decomposition

#### phase current decomposition:

Each phase: \( i_X(t) = i_{xaX}(t) + i_{uxX}(t) + i_{nxX}(t) \)

**System current decomposition:**

**RMS values:**

\[ L^2 = (i_{1a}^+)^2 + (i_{1u}^+)^2 + (i_{1u})^2 + (i_n)^2 \]

**Line loss:**

\[ P_{loss} = 3 \cdot L^2 \cdot R_L + L_n^2 \cdot R_{Ln} \]

![Diagram](image)

#### Figure 4.2.3 power loss – three-phase loss – line power loss current decomposition

### 4.2.3 three-phase loss – calculating losses by decomposing line current

The trick behind the Power Loss Calculator tool is Current Decomposition. The Unified Power Theory says that all physical phenomena like phase shift, unbalance and harmonics can be found in the instantaneous currents. If we look at the individual phase currents flowing through in-house wiring, we can separate these currents into individual values that are each the result of one of these phenomena. We call this current decomposition, and it is shown in the blue section of the diagram.

**Phase currents**

The total phase current is decomposed into an active current (the only useful one), a reactive current, an unbalance current and a non-fundamental current. Why split it into these parts? Because this will give us a good indication of where to look for improvements if we do not like the outcome of the total line loss calculation. It is one thing to conclude that a certain amount of power is lost, but another to do something about it.

- The harmonic current is again called non-fundamental because it may also include interharmonic components.
- And watch out: Capital N here again means non-fundamental while lower-case n means neutral conductor!

To calculate the total power lost in the transmission lines, we could add together the squares of the RMS values of the individual current components of each phase multiplied by the appropriate phase line resistance. This would need a lot of calculation, though, and there is a shortcut.

By defining the system current \( L \) we can quickly calculate the system-level RMS values of the phase currents by category (active, reactive, unbalance and non-fundamental). The system current is derived from
the symmetrical currents (positive-, negative- and zero sequence), which are also system currents and combine all the fundamental-frequency current phenomena. We then only have to use a separate mathematical method for the non-fundamental current.

**System current**

This system current is shown on the red section of the diagram. Be aware that this system current \( I_L \) is not a physical current, but a mathematical shortcut. It allows us to calculate the effect of the individual phase currents on the total system-level line power loss.

Once we have calculated this system current \( I_L \), calculating the total line power loss caused by the phase currents is easy: \( P_{\text{loss}} = 3 \cdot I_L^2 \cdot R_L \) in which \( R_L \) is the resistance of the local transmission line. The factor three is because we have three phase conductors contributing to the line losses. However, there is also a neutral line that can carry neutral current \( I_n \) (ideally zero). This current also causes Joule (\( I^2R \)) losses in the neutral line resistance \( R_{Ln} \) and also appears on the electricity bill.

The complete system-level line loss formula is:

\[
P_{\text{loss}} = 3 \cdot I_L^2 \cdot R_L + I_n^2 \cdot R_{Ln}
\]

We will now look at the separate parts of this system current \( I_L \).
4.2.4 three-phase loss – line current components

To find the RMS value of the system current $I_L$, we start with the RMS values of the combined individual phase currents $I_A$, $I_B$, $I_C$, and the neutral current $I_n$. ‘Combined’ means that all frequency components are included.

We next determine the RMS value of the fundamental phase current components $I_{1A}$, $I_{1B}$, $I_{1C}$. We can do this by performing a Fourier Transformation on the instantaneous phase currents $i_A(t)$, $i_B(t)$ and $i_C(t)$. We are now at level two in the blue section of the diagram. The blue section indicates we are still dealing with individual phase current values.

The next step performs the Symmetrical Transformation on the fundamental phase current values. This gives us the Symmetrical Currents $I_1^+$, $I_1^-$ and $I_1^0$. This third level is in the red section of the diagram, meaning that we lose the individual phase information and talk about values that are only meaningful at total system level.

The last step combines symmetrical and non-fundamental phase-current information to give the RMS system current components $I_L$:

$$I_L^2 = (I_{1a}^+)^2 + (I_{1r}^+)^2 + (I_{1u})^2 + (I_N)^2$$

Each of the components of the system current represents a different physical phenomenon. The system current is given by the squares of the active, reactive, unbalance and non-fundamental currents. Again, it excludes the neutral current which is processed separately.
three-phase line loss

\[
R_{\text{line}} = \frac{\rho_{\text{wire}} \cdot I_{\text{line}}}{A_{\text{wire}}} \quad (\Omega)
\]

- system line resistance:
  - \(I_{\text{line}}\): length of the line
  - \(A_{\text{wire}}\): cross area of the wire
  - \(\rho_{\text{wire}}\): specific wire resistance

- estimation: \(R_{\text{line}} = 1\% \text{ of } \frac{P_{\text{nom}}}{I_{\text{nom}}^2}\)
  this assumes each line is laid out in such a way that at maximum rated load the loss in each phase is \(\approx 1\%\) of nominal power.

Figure 4.2.5  power loss – three-phase loss – line power loss line resistance

4.2.5  three-phase loss – finding or estimating line resistance

To calculate the loss in the lines, we need to know the line resistance. To calculate this, we need to know the line length between load and the metering point, the cross sectional area of these lines and the specific wire resistance.

The length must be read from a wiring plan, or estimated. The cross sectional area of the line is often standardized depending on the maximum permissible load, and is given in \(\text{mm}^2\) or AWG units. The specific wire resistance depends on the conductor material, usually copper or aluminum. Once we have all these figures we can calculate the line resistance \(R_{\text{line}}\). The three-phase conductors almost always have equal resistance, but the neutral conductor can be different and has to be calculated separately.

What if we cannot find all the necessary information? Then we assume that the system was built following prescribed design rules with line resistances that limit the maximum loss in the lines to < 1% at maximum load. With this assumption, we can estimate the line resistances. As we shall see in our final practical example, the ratio between the various loss components is often more important than the absolute value. For a cost estimation of the power lost, however, we need accurate line resistance values.
Figure 4.2.6  power loss – three-phase loss – line power loss components

### 4.2.6  three-phase loss – calculating line power loss

Once we have determined the system current $I_L$, the neutral current $I_n$ and the line resistances $R_L$ and $R_{Ln}$, we can calculate the various losses from the different physical phenomena.

The line loss due to the **active current** is unavoidable and can only be reduced by using thicker lines, which is not always economically feasible.

The loss due to **reactive current** can be reduced by reducing the phase shift between fundamental voltage and current, for instance by using compensating capacitors.

The loss due to **non-fundamental currents** can be reduced by either avoiding generating them or by using passive or active filtering.

The loss due to **unbalance current** can be reduced by better balancing the system. This can be done by more efficient grouping of the loads or by adding passive or active balancing components.

The loss due to **neutral current** can be reduced by not generating neutral current in the first place. As this is mostly a result of harmonics and/or unbalance, filtering harmonics and balancing loads is a good start to avoid these losses.

$$P_{loss} = 3 \cdot I_L^2 \cdot R_L + I_n^2 \cdot R_{Ln}$$

$$I_L^2 = (I_{1a})^2 + (I_{1r})^2 + (I_{1u})^2 + (I_n)^2$$

$$P_{loss} = P_{Ja} + P_{Jr} + P_{JN} + P_{JU} + P_{Jn} [W]$$
4.2.7 three-phase losses – a real-world line power loss example

The diagram shows the screen of the line loss calculator tool in the Fluke 430 Series II (there are many other displays to help monitor, log and troubleshoot all kinds of power quality problems). This example was taken from the test facility of a plant manufacturing X-ray diffraction meters and spectrometers. There were 18 test stands, of which three were occupied when the measurement was taken. The tests are a 24/7 run of the X-ray machine under test, and the measurement was taken at the main feeder of the 18 test stands.

We see (bottom line) that the estimated length to the metering point is 100 m of Cu cable with 35mm\(^2\) cross sectional area, and the tariff is 17 Euro cents/kWh. The first column shows the loss categories, the second column shows the total power for these categories, the third column shows the power loss in the lines, and the fourth column the amount of money/hr involved. The bottom of the fourth column shows the total costs/year of the line power losses based on a 24/7 load.

The **active line loss** is 197 W from a total active power of 25.6 kW. This is less than 1%, which shows the in-house cabling is up to the task. **Reactive losses** are moderate but unbalance and distortion losses (non-fundamental losses) are larger than the active ones, resulting in significant neutral losses.

Even relatively small losses add up over the year. Here, the total losses running three out of 18 test stands totals almost €1700/year. For 18 out of 18 stands this would increase to over €10,000/year.

Reducing the active line losses (17.3% of the total) involves improving the energy efficiency of the equipment in the building itself. So if we subtract these we are left with a loss of about €8270/year caused by unwanted power components. In other words, we could spend €8000 on countermeasures like active filtering with a payback time of one year. The reduction in unbalance, distortion and neutral power loss costs would far outweigh the slight increase of active power consumed by the filter.
The 430 Series II also helps to identify where compensating for one type of loss impinges on another, since for example adding a capacitor to reduce reactive loss could increase harmonic loss (when the instrument would show that losses had actually increased after the corrective action). That allows you to optimize the network, otherwise difficult without extensive trial and error.

All this makes the 430 Series II a useful tool for quick scans in an industrial environment without disturbing the workflow of the application under test. It gives an overview of an application’s power efficiency in a single measurement, on a single screen and divided into the various loss categories. This allows a quick analysis of the nature of the problem. The estimated line loss costs over a year show the payback times you can expect.

What it doesn’t show, though, are the other savings you’ll make. These include avoidance of utility penalty charges from breaching power-factor limits, reduced downtime and loss of production or network from mystery faults that are hard to track down, and even equipment failures caused by poor power quality. If these start to affect customers, they will cost you much more than just money.
From the previous examples it is clear that the IEEE 1459 and Unified methods come to different values for apparent power under unbalance (and some harmonic) conditions. As a result, derived powers like $S_u$ and $S_n$ also differ. Another example will help investigate the cause of these differences. This time, we draw the supply transformer.

Consider a medium-voltage to low-voltage $\Delta$-$Y$ feeder transformer (a common configuration) with three different loads. For easy calculation, we assume the low voltage side to have a balanced 100 V RMS voltage and the loads to be 1 Ω resistors. The nominal phase current is therefore 100 A.

- **Load 1**
  This is a balanced load of three 1-Ω resistors leading to equal phase currents of 100 A and a neutral current $I_n = 0$ A. The active power $P = 30$ kW, and the IEEE 1459 and Unified values for the apparent power $S_e = S = 30$ kVA. The unbalance power $S_{eu} = S_u = 0$.

- **Load 2**
  We now introduce some unbalance by increasing the load on phase A from 1 Ω to 2 Ω. The phase-A current $I_A$ decreases from 100 to 50 A, and the neutral current $I_n$ increases to 50 A. The active power decreases to $P = 25$ kW.

  The IEEE 1459 apparent power $S_e$ also decreases, to 27.4 kVA, but not as much as the active power $P$ because part of the apparent power goes into the apparent unbalance power $S_{eu}$ of 11.2 kVA. The Unified apparent power similarly decreases, but is closer to the active power with $S = 26.0$ kVA and $S_u = 7.1$ kVA.
So far, it is hard to say which of the two results is more likely.

- **Load 3**
  
  We now remove the load on phase A, so $I_A = 0 \text{ A}$ and the neutral current $I_n$ becomes 100 A. The active power drops to $P = 20 \text{ kW}$.

  Surprisingly, IEEE 1459 calculates an apparent power of $S_e = 30 \text{ kVA}$. So load 3 is lower than load 2, but the IEEE 1459 apparent power actually increases! How is this possible? We will explain it below. The apparent IEEE 1459 unbalance power $S_u$ also rises, to 22.4 kVA.

  The Unified value of apparent power decreases as we would expect, to a somewhat lower value than in load 2 because the total load is reduced, so to $S = 24.5 \text{ kVA}$, while the unbalance component $S_u$ increases to 14.1 kVA.

So where does this different behaviour come from? It comes from the way the neutral current $I_n$ is handled in the formulae.

For IEEE 1459, the apparent power $S_e$ is calculated using the effective current $I_e^2 = (I_A^2 + I_B^2 + I_C^2 + I_n^2)$. So, for IEEE 1459, the apparent power gives the neutral current $I_n$ the same weight as the phase currents $I_A$, $I_B$ and $I_C$. That is because the IEEE 1459 apparent power is based on a balanced replacement system (effective system) that has the same line losses as the original unbalanced system. Doing it this way by definition includes the neutral current $I_n$. And that leads to overestimating transformer losses.

The Unified method uses the current $I^2 = (I_A^2 + I_B^2 + I_C^2)$ for apparent power as defined by Buchholz in 1922. It looks at the apparent power from the viewpoint of the transformer losses. The transformer losses are determined by the magnitude and unbalance of the phase currents $I_A$, $I_B$ and $I_C$ that flow through the transformer windings. The neutral current $I_n$ is on the other hand a result of the harmonics and unbalance of the phase currents $I_A$, $I_B$ and $I_C$. So the transformer losses are not caused by the neutral current itself, but by the harmonics and unbalance that generate the neutral current.

The different starting points of the two methods lead to different definitions and different values of apparent power. Which one you favour depends on your point of view. But it seems logical that, at least for derating transformers due to unbalance, the Unified method gives a more realistic figure. However, in academic circles, the debate still goes on!
Unified apparent power components

![Diagram of unified apparent power components]

5.2 Unified apparent power components

When we look at the efficiency of a power system, we are especially interested in the relation between the (useful) active power to the full (combined) apparent power. This diagram gives the relation between useful power and the power that has to be generated and transported.

This table is an extension of that just shown for the Unified approach. It builds up from the active (top) and non-active power (middle) to show the apparent power (bottom) as defined by the Unified Power Theory for different types of loads. The table introduces some non-active powers that have not yet been discussed and uses a slightly different terminology compared to IEEE terminology elsewhere used in this tutorial.

The grey symbols on the cell boundaries symbolize the relation between row and column powers.

The colored triangles show the powers that are important for different types of loads (yellow for three-phase motors, red for single-phase motors, blue for heaters/light/rectifiers, and green for general loads).

The table builds up the powers from active positive-sequence power at the top left, through the non-active powers, to the combined apparent power including all its component parts at the bottom right.

About the rows

Rows 1 – 4 list the various types of power: active, non-active, reactive as a special case of non-active, and apparent.

Looking at row 1, we have the active positive-sequence power which, added to the active unbalance power (negative- plus zero-sequence) gives the fundamental power. Add that to the distortion power and you get the combined power. Similar in row 2: add the reactive positive-sequence power to the reactive unbalance power and you get the reactive fundamental power.
So far, we have just added the individual terms, shown by the grey symbols on the cell boundaries. The relationship between the other terms is more complex (and described below).

Row 3 introduces three new terms – the non-active unbalance power $N_U$, the non-active distortion power $N_D$ and the non-active full power $N$. These non-active powers complement the relationship between the active, reactive (fundamental only) and apparent power for each of the power components. Reading the whole of row 3 shows that $Q_1^+ + Q_U$ gives the fundamental reactive power $Q_1$, and that $Q_1, N_U$ and $N_D$ combine to give the full non-active power $N$.

Row 4 is similar to row 1 but now with quadratic addition for the apparent powers.

About the columns

Columns 1 – 5 list the components of each type of power. Columns 1 – 3 show fundamental power, while 4 shows distortion and 5 shows full signal. Fundamental power contains positive-sequence power (column 1) and unbalance power (column 2) as special cases. Distortion power is called non-fundamental power in IEEE, and includes harmonic and interharmonic powers. The advantage of the word ‘distortion’ with symbol $D$ is that the symbol $N$ now only stands for Non-active rather than Non-fundamental. The full signal power, containing fundamental and distortion components, is called combined power in IEEE and in this tutorial.

Relations between powers

The grey symbols on the cell boundaries, again, symbolize the relations between the powers which are, in full:

<table>
<thead>
<tr>
<th>Power Type</th>
<th>Row</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>active powers</td>
<td>(row 1)</td>
<td>$P = P_1 + P_D$</td>
</tr>
<tr>
<td>reactive powers</td>
<td>(row 2)</td>
<td>$Q = Q_1^+ + Q_U$</td>
</tr>
<tr>
<td>nonactive powers</td>
<td>(row 3)</td>
<td>$N = N_U^2 + N_D^2 + Q_1^2$</td>
</tr>
<tr>
<td>apparent powers</td>
<td>(row 4)</td>
<td>$S = (S_1^+)^2 + S_U^2$</td>
</tr>
<tr>
<td>pos. seq. powers</td>
<td>(column 1)</td>
<td>$(S_1^+)^2 = (P_1^+)^2 + (Q_1^+)^2$</td>
</tr>
<tr>
<td>unbalance powers</td>
<td>(column 2)</td>
<td>$S_U^2 = P_U^2 + Q_U^2 + N_U^2 + 2\cdot P_1^+ \cdot Q_U + 2\cdot Q_1^+ \cdot P_1$</td>
</tr>
<tr>
<td>fund. powers</td>
<td>(column 3)</td>
<td>$S_1^2 = P_1^2 + Q_1^2 + N_U^2$</td>
</tr>
<tr>
<td>distortion powers</td>
<td>(column 4)</td>
<td>$S_0^2 = P_0^2 + N_0^2 + 2\cdot P_D \cdot P_1$</td>
</tr>
<tr>
<td>full powers</td>
<td>(column 5)</td>
<td>$S^2 = P^2 + N^2$</td>
</tr>
</tbody>
</table>

Nonactive powers

With the relations above, we can define the newly introduced non-active powers:

The non-active unbalance power $N_U$ is defined as: $N_U^2 = S_U^2 - P_1^2 - Q_1^2$.

The non-active distortion power $N_D$ is defined as: $N_D^2 = N^2 - Q_1^2 - N_U^2$.

The non-active full power $N$ is defined as: $N^2 = S^2 - P^2$.

Notice that there is no such thing as reactive distortion power or reactive full power, and all positive-sequence non-active power is reactive power.

Row relations

As we have seen, the active power components in row 1 can simply be added. They all work along the x-axis in the P-Q-S diagram. The same is true for the reactive power components in row 2, working along the y-axis. These relations are all linear. For the other non-active power components in row 3 and the apparent
power components in row 4, the relation is quadratic because they contain components working in different directions.

**Column relations**

The positive-sequence powers in column 1 have the quadratic relation we have seen before in the P-Q-S diagram. The unbalance powers in column 2 are more complicated and contain cross power components. The fundamental powers in column 3 have a quadratic relation. The distortion powers in column 4 contain cross power components again. The full powers in column 5 have a quadratic relation.

**Load dependant analysis**

The table shows the relation between useful active power and apparent power for different types of loads:

**three-phase motors**

The only useful power for a three-phase motor is the positive-sequence active power \( P_1^+ \). \( P_U \) and \( P_D \) are real losses and could even damage the motor. Nevertheless customers pay for \( P_U \) and \( P_D \) as well as for \( P_1^+ \). The relation between useful power \( P_1^+ \) and total delivered power \( S \) is shown in:

\[
S^2 = (P_1^+)^2 + (Q_1^-)^2 + S_U^2 + S_D^2
\]

\( Q_1^- \) (phase shift), \( S_U \) (unbalance) and \( S_D \) (distortion) characterize the losses.

**single-phase motors**

For three single phase motors on a three-phase network the useful power is the fundamental active power \( P_1 \). Here the relation between useful power \( P_1 \) and total delivered power \( S \) is shown in:

\[
S^2 = P_1^2 + Q_1^2 + N_U^2 + S_D^2
\]

\( Q_1 \) (phase shift), \( N_U \) (unbalance) and \( S_D \) (distortion) characterize the losses.

**heating, lighting and rectifiers**

For these type of loads all active power will likely be used so the useful power will be the full active power \( P \). Here the relation between useful power \( P \) and total delivered power \( S \) is shown in:

\[
S^2 = P^2 + Q_1^2 + N_U^2 + N_D^2
\]

\( Q_1 \) (phase shift), \( N_U \) (unbalance) and \( N_D \) (distortion) characterize the losses.

**general loads**

If we know which components of the active power are turned into useful power for a particular load, we can use the power relation formulae from the table to write a single formula giving the relation between useful power, total delivered power and wasted power. The most general relation is given by:

\[
S^2 = P^2 + N^2
\]

the full nonactive power \( N \) characterizes the combined losses. Because of its general nature the formula does not reveal much on the nature of the losses (phase shift, unbalance or distortion).

To analyze system efficiency we need to know which components of the active power are turned into useful power by the system load. We can than write a formula showing the relation between useful power, total delivered power and wasted power.
6 electrical timeline

Up to 1870s – basic electrical and magnetic theory

600 BC Thales discovered static electricity
1600 William Gilbert coined the words electricity, magnetic pole and electric force
1752 Benjamin Franklin demonstrated that lightning was electricity
1800 Alessandro Volta developed first electric battery and first electrical circuit
1820 Hans Christian Ørsted discovered electromagnetism
1821 Michael Faraday demonstrated electromagnetic fields and invented electric motor
1825 André-Marie Ampère related electrical to magnetic fields
1827 Georg Ohm Ohm’s Law: the basic electrical equation: U = I . R
1832 Hippolyte Pixii First (hand-cranked) AC generator
1841 James Joule Joule’s Law of electrical heating: power P = I^2 . R
1845 Gustav Kirchhoff Kirchhoff’s laws: (1) the sum of currents into a node equals those out of it, and (2) the directed sum of voltages around a closed loop is 0
1873 James Maxwell found equations for e-m fields to unify electricity and magnetism

1870s to 1890s – practical electrical power transmission

1873 Hippolyte Fontaine first DC (Direct Current) transmission (Vienna World Exhibition)
1879 Thomas Edison first reliable, economic electric light bulb
1882 Thomas Edison first DC power stations in Appleton (later New York and Paris).
1883 Thomas Edison first three-wire transmission
1884 Gaulard first single-phase AC (Alternating Current) transmission (Turin Exhibition)
1885 Charles Parsons first to generate electricity using a steam turbine
1888 Nikola Tesla invented the rotating-field AC induction motor
1888 Hermann Aron first accurate Watt-hour meter
1890 Westinghouse introduced 60 cycle AC in US
1891 Mikhail Dobrovolsy first long-distance (175 km) three-phase AC transmission (Frankfurt)
1891 AEG introduced 50 cycle AC in Europe
1893 André Blondel in any system of electrical conductors you need one fewer power meter than you have wires

1890s to 1990s – developing AC power analysis

1897 Charles Steinmetz showed that complex quantities can represent AC waveforms
1918 Charles Fortesque introduced the idea of symmetrical components
1922 Max Buchholz defined reactive and apparent powers in three-phase systems
1927 C. I. Budeanu developed reactive and distortion power theory
1932 S. Fryze separated non-sinusoidal currents into active/reactive components
1935 IEEE Dictionary defined single- and three-phase, linear and nonlinear systems
1935 W. V. Lyon developed apparent power and related quantities
1972 Shepherd and Zakikhani developed theory for reactive power in single-phase circuits
1980 Kusters and Moore extended reactive power theory under non-sinusoidal conditions
1981 Z. Nowomiejski generalized theory of electrical power
1987 Leszek Czarnecki decomposed single-phase currents under distortion
1997 Vincente León unified several competing power theories into the Unified model
1999 Alexander Emanuel chaired IEEE 1459, defined effective apparent power in unbalanced systems.
7 references


Budeanu, C.I. “Puissances reactives et fictives”. Institut Romain de L'Energie, Bucharest - 1927.


